MATHSAURUS

Pearson Edexcel International GCSE Mathematics A (4MA1)

November 2024 Paper 2H

Higher Tier · Calculator permitted

Mark schemes you can actually read.

Students always tell me the official mark schemes are hard to read. That's because those documents are written for examiners and teachers, not for the people actually sitting the exam. So I've fixed that. I've taken the official mark scheme, the worked solutions, and the most useful parts of the examiner's report, and turned them into one document written for you. Every step is explained, every mark is labelled, and I've highlighted what real students got wrong on this paper, so you don't have to read the full report yourself to learn from it. I really hope it helps you with your studies. If it does, there's a lot more at mathsaurus.com: courses, video tutorials, and resources I've spent years building up. Have a look once you're done.

Dr Kevin Olding, creator of mathsaurus.com

Question index *(click the links to jump to that question)*

How to read this document

These worked solutions are designed as an *expanded mark scheme*. We show every step of the working, explain the reasoning, and indicate exactly where each mark is awarded. There are three things to look out for as you read.

Mark annotations

Inline labels like **[M1]**, **[A1]** and **[B1]** appear next to the working step where each mark is earned. These follow the official Edexcel mark-scheme notation:

- **[M1]**, **[M2]**, ... **Method marks.** Awarded for using a correct method, even if the arithmetic later goes wrong. You earn these by writing down the right approach.
- **[A1]**, **[A2]**, ... **Accuracy marks.** Awarded for the correct value or expression. An A mark almost always depends on having earned the related M mark first.
- **[B1]**, **[B2]**, ... **Unconditional accuracy marks.** Awarded for a correct answer that doesn't depend on any method mark (e.g. writing down a single value).

You will also see these abbreviations:

- **cao**, *correct answer only.* No follow-through allowed: the value must be exactly right.
- **ft**, *follow through.* A method applied correctly to your own earlier (possibly wrong) values can still earn the mark.
- **oe**, *or equivalent.* The answer can take any equivalent form (e.g. $\frac{7}{5}$ or 1.4 or $1\frac{2}{5}$).
- **dep**, *dependent.* You can only earn this mark if you have earned an earlier specified mark.
- **isw**, *ignore subsequent working.* Once you have reached the answer, harmless extra working will not lose you marks.

Mathsaurus tips (blue boxes)

Blue boxes give you the *why*: the strategy behind a method, an insight that makes a hard step easy, or a common trap and how to avoid it. Each tip carries its own short label so you can scan for the one you need.

Notes from the examiner's report (yellow boxes)

Yellow boxes flag what real students got wrong on each question, drawn from the official Edexcel examiner's report. We give a brief, paraphrased note here rather than the full report; the complete examiner's commentary for this paper is available on the Pearson qualifications website.

Question 1 Multiplying mixed-number fractions (3 marks)

Show that $1\frac{5}{7} \times 2\frac{3}{16} = 3\frac{3}{4}$.

Step 1: convert each mixed number to an improper fraction. **[M1 (for both fractions written as improper fractions)]**

$$1\tfrac{5}{7} = \frac{12}{7}, \qquad 2\tfrac{3}{16} = \frac{35}{16}.$$

Step 2: multiply, cancelling where possible. Spot the common factors before multiplying: 12 and 16 share a factor of 4, and 35 and 7 share a factor of 7. **[M1 (for multiplying numerators and denominators, or cancelling)]**

$$\frac{12}{7} \times \frac{35}{16} = \frac{12 \times 35}{7 \times 16} = \frac{3 \times 5}{1 \times 4} = \frac{15}{4}.$$

Step 3: convert back to a mixed number to match the given form. **[A1 (completion to given result, dep on M2)]**

$$\frac{15}{4} = 3\tfrac{3}{4}. \qquad \boxed{1\tfrac{5}{7} \times 2\tfrac{3}{16} = 3\tfrac{3}{4}}$$

A fully correct one-line solution would also score full marks:

$$1\tfrac{5}{7} \times 2\tfrac{3}{16} = \frac{12}{7} \times \frac{35}{16} = \frac{12 \times 35}{7 \times 16} = \frac{420}{112} = \frac{15}{4} = 3\tfrac{3}{4}.$$

The cancelling approach above is more efficient, but a less efficient method that's fully correct (e.g. multiplying first then simplifying 420/112) is still worth full marks.

Mathsaurus tip: What "Show that" means

"Show that" tells you the answer up front and asks you to write the working that gets there. The marks are for the steps, not the final value. Two things are essential: every line of working must be visible (so the examiner can follow your reasoning), and the working must be *fully* correct (a fluke from incorrect working scores zero, even if the answer matches). On a non-calculator question, cancelling early keeps numbers small, but multiplying out first and simplifying at the end is equally valid.

Notes from the examiner's report

Many students gained full marks. The most common slip was jumping straight from the improper fractions to 15/4 on a calculator without showing the intermediate step. On a "show that" question you must write down the working. A few students arrived at the right answer from incorrect improper fractions, which scored zero: if you don't reach the given answer cleanly, re-check your conversions.

Total: M1 + M1 + A1 = 3 marks.

Question 2 Upper and lower bounds (2 marks)

A length is measured as 1.4 metres, correct to one decimal place. The actual value lies anywhere in the interval that rounds to 1.4.

(a) Upper bound. The largest value that still rounds to 1.4 at 1 d.p. sits halfway between 1.4 and 1.5:

$$\boxed{1.45 \text{ m}}.$$

[B1 (allow $1.44\dot{9}$ or $1.44999\ldots$)]

(b) Lower bound. The smallest value that still rounds to 1.4 at 1 d.p. sits halfway between 1.3 and 1.4:

$$\boxed{1.35 \text{ m}}.$$

[B1 cao]

Mathsaurus tip: Be careful with $1.44\dot{9}$ and 1.449

The official mark scheme also accepts $1.44\dot{9}$ (i.e. $1.4499\ldots$ recurring) for the upper bound. That's because $1.44\dot{9} = 1.45$ *exactly* — a recurring 9 always equals the next digit up. So mathematically the two answers are the same number.
The trap: don't write 1.449 or 1.4499 (terminating decimals). Those are strictly less than 1.45, so they're outside the bound. The safest answer is just 1.45.

Notes from the examiner's report

Most students gave 1.45 and 1.35 correctly. A small minority showed no understanding at all, writing things like $1.4 \times 2 = 2.8$ or $1.4 \div 2 = 0.7$. Common wrong attempts included 1.44, $1.4 + 0.5 = 1.9$, $1.4 - 0.5 = 0.9$, or 1.5 and 1.3.

Total: B1 + B1 = 2 marks.

Question 3 Right-angled trigonometry (3 marks)

Triangle PQR has a right angle at Q, hypotenuse $PR = 8.6$ cm, and angle $PRQ = 43°$. We want $x = QR$.

Looking from the $43°$ angle: QR is the *adjacent* side and PR is the *hypotenuse*. So use cosine: **[M1 (for a correct trig statement for x)]**

$$\cos 43° = \frac{x}{8.6}.$$

Rearrange: **[M1 (for a fully correct calculation to find x)]**

$$x = 8.6 \cos 43° \approx 6.290\ldots$$

To 1 d.p.:

$$\boxed{x = 6.3 \text{ cm}}.$$

[A1 (awrt 6.3)]

Mathsaurus tip: SOH CAH TOA in one breath

Label the sides relative to the angle you know: opposite, adjacent, hypotenuse. Then pick the ratio that uses the two sides you have. Here the $43°$ angle has QR adjacent and PR as the hypotenuse, so cos is the right choice.

Notes from the examiner's report

Higher-tier students did this very well, almost all using cosine as expected. Some chose a longer route via sine and then Pythagoras, or even via the sine rule. Two-stage methods are more prone to rounding errors. A few students found the opposite side instead of the adjacent.

Total: M1 + M1 + A1 = 3 marks.

Question 4 Reverse percentage and percentage change (5 marks)

(a) 17% of N is 357. Find N.

Set up the equation directly: **[M1 (for a correct calculation or equation in N)]**

$$0.17 \times N = 357 \quad \Longrightarrow \quad N = \frac{357}{0.17}.$$

$$N = \boxed{2100}.$$

[A1 cao]

(b) Population went from 650 in 2019 to 806 in 2020. Find the percentage increase.

Find the actual increase first: **[M1 (for $806 - 650 = 156$ or $806/650 = 1.24$)]**

$$\text{increase} = 806 - 650 = 156.$$

Express the increase as a percentage of the *original* value: **[M1 (for a correct calculation for the percentage increase, or seeing 124 or 0.24)]**

$$\frac{156}{650} \times 100 = 24\%.$$

$$\boxed{24\%}.$$

[A1 cao]

Mathsaurus tip: Percentage change always uses the original

The denominator for any percentage change is always the *original* value, not the new one. Dividing by the new value (here 806) gives a slightly smaller percentage and is one of the most common slips on this topic.

Notes from the examiner's report

(a) The standard slip was finding 17% of 357 rather than the other way round, then multiplying or dividing by 0.83 or 1.17. (b) Many gained full marks. Some left the answer as 124% or 0.24% (forgetting to subtract 1 or to multiply by 100). The most common method error was dividing the difference by 806 instead of 650.

Total: M1+A1 + M1+M1+A1 = 5 marks.

Question 5 Probability and expected frequency (4 marks)

Probabilities given: $P(2) = 0.14$, $P(3) = 0.17$, $P(5) = 0.21$. Also $P(1) = P(4)$. Cody spins 400 times.

Step 1: probabilities sum to 1. **[M1 (for use of probabilities total 1)]**

$$P(1) + P(2) + P(3) + P(4) + P(5) = 1.$$

So:

$$P(1) + P(4) = 1 - (0.14 + 0.17 + 0.21) = 1 - 0.52 = 0.48.$$

Step 2: split equally between 1 and 4. **[M1 (for a complete method to find $P(4)$)]**

$$P(4) = \frac{0.48}{2} = 0.24.$$

Step 3: multiply by 400 for the expected count. **[M1 (for 0.24×400)]**

$$0.24 \times 400 = \boxed{96}.$$

[A1 cao]

Mathsaurus tip: Expected frequency

Expected frequency is just probability multiplied by the number of trials. The probability tells you the long-run fraction of spins that land on 4, so over 400 spins you'd expect $0.24 \times 400 = 96$.

Notes from the examiner's report

Many students gained full marks. The two most common errors: forgetting to subtract from 1 when finding the missing probabilities, and stopping at 0.24 as if that were the final answer. Some answered 192 because they didn't halve the 0.48 between the two equal probabilities.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 6 Surface area of a triangular prism (3 marks)

The triangular cross-section has legs 6 cm and 8 cm with hypotenuse 10 cm (a $6, 8, 10$ right triangle). The prism length is 15 cm. The surface area is the sum of five faces: two triangular ends and three rectangular sides.

Two triangular ends: **[M1 (for a correct method to find the areas of 2 different faces)]**

$$2 \times \frac{1}{2} \times 8 \times 6 = 2 \times 24 = 48 \text{ cm}^2.$$

Three rectangles, each with length 15 cm:

$$15 \times 8 = 120, \qquad 15 \times 6 = 90, \qquad 15 \times 10 = 150 \text{ (the slant face)}.$$

Add them all up: **[M1 (for adding 4 or 5 values, at least 3 from a correct method)]**

$$48 + 120 + 90 + 150 = \boxed{408 \text{ cm}^2}.$$

[A1 cao]

Mathsaurus tip: Don't miss the slant face

A triangular prism has **five** faces: 2 triangles and 3 rectangles. The slant face (the rectangle along the hypotenuse) is the one students forget most often. Imagine unfolding the prism flat and you'll see all five.

Notes from the examiner's report

Quite a lot of full marks, but more confusion than expected on a fairly standard question. Common slips: finding the volume rather than the surface area, forgetting to halve when finding the triangle area, missing the slant face, or missing one of the triangular ends. Some students even brought π into it. A few used Pythagoras to "find" the hypotenuse as 10, despite it already being given on the diagram.

Total: M1 + M1 + A1 = 3 marks.

Question 7 Drawing lines and shading a region (4 marks)

(a) Draw the three lines on the grid:

- **(i)** $x = 3$: a vertical line through $x = 3$. **[B1]**
- **(ii)** $y = 1$: a horizontal line through $y = 1$. **[B1]**
- **(iii)** $x + y = 7$: a straight line. The easiest way to plot it is to find two points: when $x = 0$, $y = 7$; when $y = 0$, $x = 7$. Join $(0, 7)$ to $(7, 0)$. **[B1]**

(b) The region R satisfies $x \geq 3$, $y \geq 1$, and $x + y \leq 7$.

This is a triangular region bounded by:

- $x = 3$ on the left,
- $y = 1$ on the bottom,
- $x + y = 7$ on the upper right.

The three corner points are $(3, 1)$, $(6, 1)$, and $(3, 4)$. Shade this triangle and label it R. **[B1 (correct region shaded and labelled R)]**

Either convention is fine: shade the region R itself, or shade everything outside *R. As long as the region inside the triangle is clearly labelled R, the mark is awarded.*

Mathsaurus tip: Test a point if you're not sure which side

For each inequality, after drawing the boundary line, the shaded region is one side of it. If you can't tell which side "$\geq$" or "$\leq$" picks out, just *test a single point* (the origin $(0, 0)$ usually works if it's not on the line). Substitute it into the inequality: if it's true, that side is included; if it's false, the other side is. Here for $x + y \leq 7$, testing $(0, 0)$ gives $0 \leq 7$ which is true, so the origin's side of the line is the one we want.

Notes from the examiner's report

Generally well attempted. Common slips: mixing up $x = 3$ with $y = 3$, drawing the line $x + y = 7$ incorrectly (sometimes joining $(0, 7)$ to $(8, 0)$ by accident, or drawing a positive-gradient line). For (b), students who got at least one vertical line, one horizontal line, and one negative-gradient line usually picked up the shading mark on follow-through.

Total: B1 + B1 + B1 + B1 = 4 marks.

Question 8 Mean, working backwards (3 marks)

Kim's 4 bananas have mean 145 g, so their total weight is:

$$4 \times 145 = 580 \text{ g}.$$

[M1 (for one correct product)]

After Andy adds his banana, the 5 bananas have mean 142 g, so their total weight is:

$$5 \times 142 = 710 \text{ g}.$$

The new banana's weight is the difference: **[M1 (for a fully correct method to find the weight of the 5th banana)]**

$$710 - 580 = \boxed{130 \text{ g}}.$$

[A1 cao]

Mathsaurus tip: Mean = total ÷ count

Whenever you're given a mean and a count, multiply them to get the total. Once everything is in totals (rather than means), addition and subtraction work as you'd expect. This trick comes up again and again.

Notes from the examiner's report

Many gained full marks. Wrong attempts often divided 145 by 4 and 142 by 5, then did unpredictable things with 36.25 and 28.4. A few students misread the question as "the total weight of the 4 bananas is 145". Some who got the right answer of 130 then divided by 5 at the end, losing the final mark.

Total: M1 + M1 + A1 = 3 marks.

Question 9 Compound interest (3 marks)

Nisha invests €20 000 for 3 years at 3.5% per year compound interest.

The compound-interest multiplier for one year is $1 + \frac{3.5}{100} = 1.035$. Apply it three times: **[M1 (for finding** 103.5% **or** 3.5% **of** 20 000**)] [M1 (for a complete method)]**

$$20\,000 \times 1.035^3.$$

Compute $1.035^3 = 1.108717\ldots$:

$$20\,000 \times 1.108717\ldots = 22\,174.34\ldots$$

To the nearest euro:

$$\boxed{€\,22\,174}.$$

[A1 (allow 22 174 – 22 175**)]**

Mathsaurus tip: Direct multiplier vs. step-by-step

The single-line method ($20\,000 \times 1.035^3$) is faster and avoids rounding. The step-by-step method ($20\,700 \to 21\,424.50 \to \ldots$) gives the same answer, but each step is another chance to round and lose accuracy.

Notes from the examiner's report

A good source of 3 marks for many. Some used simple interest ($3.5\% \times 20\,000 \times 3$) and gained just 1 mark. The biggest cause of 0 marks was using 0.35 instead of 0.035 for the rate. Some cubed 0.035 instead of 1.035. Convert percentages to decimals carefully.

Total: M1 + M1 + A1 = 3 marks.

Question 10 Pie chart and algebra combined (5 marks)

Year 11. The numbers add to 320: **[M1 (for a correct equation in x)]**

$$(3x+6)+(5x+8)+(7x-9)=320 \implies 15x+5=320.$$

Solve: **[A1 (for $x=21$ or $3x=63$)]**

$$15x=315 \implies x=21.$$

So Year 11 Biology $=3(21)+6=69$ students. **[M1 ft (for $3\times21+6=69$, dep on M1)]**

Year 10. The Biology angle is 126° out of 360°, applied to 300 students: **[M1 (independent; for a correct method for Year 10 Biology)]**

$$\frac{126}{360}\times300=105.$$

Difference:

$$105-69=\boxed{36}.$$

[A1 cao (dep on previous A1)]

Mathsaurus tip: Angle is a fraction of 360, not of the total

On a pie chart, every angle is a fraction of 360°, never of the number of items. So Biology $=\frac{126}{360}$ of the Year 10 cohort. Multiply by the cohort size (300) to get the count.

Notes from the examiner's report

Working was generally well set out. A common mark of 3 came from students who handled the Year 11 algebra cleanly but stumbled on the pie chart. Many divided 126 by 300 instead of 360. Some thought the angle equalled the number of students directly. A few even tried to link the two year groups, writing things like $126=3x+6$.

Total: M1 + A1 + M1 + M1 + A1 = 5 marks.

Question 11 Angles in a pentagon and hexagon (5 marks)

$ABCDE$ is a regular pentagon and $DEFGHI$ is a regular hexagon, sharing the edge DE. We are told AF is a straight line.

The interior angle of a regular polygon with n sides is

$$\frac{(n-2)\times 180^\circ}{n}.$$

Step 1: interior angle of the pentagon at E. With $n = 5$: **[M1]**

$$\frac{(5-2)\times 180}{5} = 108^\circ.$$

Step 2: interior angle of the hexagon at E. With $n = 6$: **[M1]**

$$\frac{(6-2)\times 180}{6} = 120^\circ.$$

Step 3: angle AEF at point E. The angles at E in the pentagon, the hexagon, and the triangle AEF together make a full turn (360°): **[M1 (a fully correct method to find angle AEF)]**

$$\angle AEF = 360^\circ - 108^\circ - 120^\circ = 132^\circ.$$

Step 4: triangle AEF is isosceles. EA is a side of the pentagon and EF is a side of the hexagon. Both regular polygons share the edge DE, so they have the same side length. So $EA = EF$, making triangle AEF isosceles. The two base angles are equal: **[M1 (a fully correct method to find angle EAF)]**

$$\angle EAF = \frac{180^\circ - 132^\circ}{2} = \boxed{24^\circ}.$$

[A1 cao]

Mathsaurus tip: Spot the isosceles triangle

The key step here is realising that $EA = EF$. Both come from regular polygons that share a side, so all the sides are the same length. Without that, you can't finish the question. Looking for equal lengths whenever regular shapes share an edge is a habit worth building.

Notes from the examiner's report

A good source of marks for many, but plenty found it tricky. The most common slip was mixing up interior and exterior angles. Many students reached $\angle AEF = 132^\circ$ for the first three marks and stopped there, not realising that triangle AEF is isosceles. Marking the angles you find on the diagram is a strong habit on questions like this one.

Total: M1 + M1 + M1 + M1 + A1 = 5 marks.

Question 12 Algebraic fractions and changing the subject (7 marks)

(a) Solve $\dfrac{3x+2}{5} - \dfrac{2x+1}{3} = x$.

Multiply every term by 15 (the common denominator) to clear the fractions: **[M1 (for writing fractions over a common denominator or removing the denominator)]**

$$3(3x+2) - 5(2x+1) = 15x.$$

Expand carefully (watch the signs on the second bracket): **[M1 (for an equation with no brackets or fractions)]**

$$9x + 6 - 10x - 5 = 15x \quad \Longrightarrow \quad -x + 1 = 15x.$$

Collect x terms:

$$1 = 16x \quad \Longrightarrow \quad \boxed{x = \tfrac{1}{16}}.$$

[A1 oe (e.g. 0.0625, dep on M1)]

(b) Make c the subject of $f = \sqrt{\dfrac{a+bc}{c-d}}$.

Step 1: square both sides. [M1]

$$f^2 = \frac{a+bc}{c-d}.$$

Step 2: multiply both sides by $(c-d)$ and expand. [M1]

$$f^2(c-d) = a + bc \quad \Longrightarrow \quad cf^2 - df^2 = a + bc.$$

Step 3: collect c on one side. The c appears twice, so factorise: **[M1 (for isolating terms in c on one side)]**

$$cf^2 - bc = a + df^2 \quad \Longrightarrow \quad c(f^2 - b) = a + df^2.$$

Step 4: divide.

$$\boxed{c = \frac{a + df^2}{f^2 - b}}.$$

[A1 oe]

Mathsaurus tip: Two-c trap

Whenever the variable you're solving for appears in both the numerator and the denominator, you'll need to gather both copies on the same side and factorise. Trying to isolate c without factorising leads to c being on both sides at the end, which is no good.

Notes from the examiner's report

(a) Many gained full marks; sign slips when expanding $-5(2x+1)$ were the most common error. Some students forgot to multiply the x on the right-hand side by the common denominator. A few lost the x on the right entirely and treated it as zero. (b) A typical changing-the-subject question. Most squared both sides correctly. The standard mistake was failing to factorise (i.e. not realising c appears twice). Sign errors and misreading their own writing also cost marks.

Total: M1 + M1 + A1 + M1 + M1 + M1 + A1 = 7 marks.

Question 13 Cumulative frequency (6 marks)

(a) Cumulative frequencies (running totals): **[B1 (for $4, 19, 39, 50, 56, 60$)]**

$$4, \quad 4+15=19, \quad 19+20=39, \quad 39+11=50, \quad 50+6=56, \quad 56+4=60.$$

(b) Plot at the **upper end** of each class: $(1,4), (2,19), (3,39), (4,50), (5,56), (6,60)$, and start the curve from $(0,0)$ since no parcel weighs less than 0. Join with a smooth curve through all seven points. **[B2 (B1 if 5 or 6 points correct but not joined as a curve)]**

(c) Median. The median is the value at cumulative frequency 30. Read across from 30 on the vertical axis to your curve, then down to the weight axis. **[B1 ft (any value in range, ft an increasing graph)]**

$$\text{median} \approx \boxed{2.5 \text{ kg}} \quad \text{(accept any value in 2.3–2.7).}$$

(d) Number of parcels weighing more than 3.7 kg. Read up from 3.7 to your curve, across to the cumulative frequency axis. You should get a value somewhere between 45 and 48. The number that weigh *more* than 3.7 is everything above that, so subtract from 60: **[M1 (for a correct reading at 3.7 kg, e.g. $45-48$)]**

$$60 - (\text{reading}) \approx 60 - 46 = 14.$$

So:

$$\boxed{14 \text{ parcels}} \quad \text{(accept any whole number in } 12, 13, 14, 15).$$

[A1 ft (must be a whole number; ft if a correct cf graph is drawn)]

Mathsaurus tip: Plot at the upper end of each class

On a cumulative frequency curve, each point goes at the *upper* end of its class, because the running total is the count of everything up to that value. Plotting at midpoints gives a graph that's shifted half a class width too far left.

Notes from the examiner's report

A good source of marks. Common slips: plotting at mid-interval or lower-interval values rather than upper ends; reading 30 on the weight axis instead of the cumulative frequency axis when finding the median; giving 3 as the median (the halfway point of the x-axis); reading the count of parcels at most 3.7 kg rather than more than. Students who showed their reading on the graph often picked up the M1 even with a wrong final answer.

Total: B1 + B2 + B1 + M1 + A1 = 6 marks.

Question 14 Index laws and solving (3 marks)

Given $\dfrac{3^{2n+3}}{3^4} = 3^3 \times 3^{1-2n}$, find n.

Left-hand side: use $\dfrac{a^m}{a^n} = a^{m-n}$. **[M1 (for one rule of indices used correctly)]**

$$\frac{3^{2n+3}}{3^4} = 3^{2n+3-4} = 3^{2n-1}.$$

Right-hand side: use $a^m \times a^n = a^{m+n}$.

$$3^3 \times 3^{1-2n} = 3^{3+1-2n} = 3^{4-2n}.$$

So the equation becomes:

$$3^{2n-1} = 3^{4-2n}.$$

Equate the powers (since the bases match): **[M1 (for a correct equation in n without indices)]**

$$2n - 1 = 4 - 2n.$$

Solve:

$$4n = 5 \quad \Longrightarrow \quad \boxed{n = \tfrac{5}{4}}.$$

[A1 oe (dep on M1)]

Mathsaurus tip: Same base, equate the powers

If $a^P = a^Q$ for any base $a > 0, a \neq 1$, then $P = Q$. The whole technique here is to get both sides into the form (a single power of 3), then drop the bases.

Notes from the examiner's report

Several students gave fully correct working. A frustrating mistake: getting to $2n - 1 = 4 - 2n$ correctly, then writing $4n = 3$ instead of $4n = 5$. Some students multiplied the powers instead of adding when combining $3^3 \times 3^{1-2n}$. Others wrote the LHS as $\frac{2n+3}{4}$ rather than $2n + 3 - 4$. Powers of 9, 27 and even 81 also turned up.

Total: M1 + M1 + A1 = 3 marks.

Question 15 Recurring decimal to a fraction (2 marks)

Show that $0.7\dot{6}\dot{3} = \frac{42}{55}$. (The dots over the 6 and 3 mean the block "63" repeats.)

So $0.7\dot{6}\dot{3} = 0.7636363\ldots$

Step 1: pick two multiples of x that line up the recurring part. Let $x = 0.7\dot{6}\dot{3}$. **[M1 (for two recurring decimals which when subtracted give a whole/terminating number)]**

$$1000x = 763.636363\ldots, \qquad 10x = 7.636363\ldots$$

Step 2: subtract to remove the repeating tail.

$$1000x - 10x = 763.636363\ldots - 7.636363\ldots$$

$$990x = 756.$$

Step 3: simplify the fraction. **[A1 (for completion to $42/55$, dep on M1)]**

$$x = \frac{756}{990}.$$

Divide top and bottom by 18 (or by 2 then 9):

$$\frac{756}{990} = \frac{42}{55}. \qquad \boxed{0.7\dot{6}\dot{3} = \tfrac{42}{55}}$$

Mathsaurus tip: Match the recurring tail

The point of multiplying by 10 and 1000 (rather than, say, 10 and 100) is that both versions then have the *same* digits after the decimal point ($\ldots636363\ldots$). When you subtract, that tail vanishes and you're left with a whole number on the right.

Notes from the examiner's report

Many examples of good working. However, some students knew they needed $100x$ or $10x$ but had no understanding of how to use them, and some did not use algebra at all (capped at 1 mark, since the question explicitly says "use algebra"). Others scored $M1$ but didn't show the fraction $756/990$ before simplifying. A few used incorrect multiplier combinations and scored zero.

Total: M1 + A1 = 2 marks.

Question 16 Circle theorems (5 marks)

A, B, C, D lie on a circle, centre O. The diagram gives $\angle ACD = 54°$ and $\angle ODC = 28°$.

(a)(i) Angle AOD.

The chord AD subtends $\angle ACD$ at the circumference and $\angle AOD$ at the centre. The **angle at the centre is twice the angle at the circumference**:

$$\angle AOD = 2 \times 54 = \boxed{108°}.$$

[B1]

(a)(ii) Reason: **the angle at the centre is twice the angle at the circumference**. **[B1 (dep on (a)(i) correct)]**

(b) Angle CAO.

Look at the quadrilateral $ACDO$. The interior angle at O is the *reflex* of $\angle AOD$ (going from D back round to A the long way), namely $360° - 108° = 252°$. The interior angles at C and D are the marked $54°$ and $28°$.

Angles in any quadrilateral sum to $360°$:

$$\angle CAO = 360° - 252° - 54° - 28° = \boxed{26°}.$$

[B1]

(c) Angle ABC.

$ABCD$ is a cyclic quadrilateral, so opposite angles sum to $180°$. We need $\angle ADC$.

Triangle OAD is isosceles (OA and OD are both radii), so the base angles are equal: **[M1 ft (using their value of (a)(i))]**

$$\angle ODA = \frac{180° - 108°}{2} = 36°.$$

The angle at D in the cyclic quadrilateral is the full angle from DA to DC, which splits at O:

$$\angle ADC = \angle ODA + \angle ODC = 36° + 28° = 64°.$$

By the cyclic-quadrilateral rule:

$$\angle ABC = 180° - \angle ADC = 180° - 64° = \boxed{116°}.$$

[A1 cao]

Mathsaurus tip: Mark every angle as you find it

Once $\angle AOD = 108°$ is on the diagram, the isosceles triangle OAD gives $36°$ at both A and D, and from there every other angle is just addition and subtraction.

Notes from the examiner's report

Most students got angle AOD, though some gave inadequate reasons such as "it is double 54" or "kite theorem". The full statement (angle at the centre is twice the angle at the circumference) must be there, with the underlined keywords. Part (b) tripped some students who thought $\angle ACO = \angle CDB$. Part (c) was a good discriminator: misconceptions included assuming $90°$ angles were present, or adding the marked angles and subtracting from 180 (showing a misunderstanding of the cyclic-quadrilateral rule).

Total: B1 + B1 + B1 + M1 + A1 = 5 marks.

Question 17 Magnitude of a vector (3 marks)

We have $\overrightarrow{FG} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\overrightarrow{HG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$.

Step 1: build $\overrightarrow{HF}$ from the given vectors. Travel from H to G, then from G to F (i.e. reverse $\overrightarrow{FG}$): **[M1 (for a correct calculation for $\overrightarrow{HF}$ or $\overrightarrow{FH}$)]**

$$\overrightarrow{HF} = \overrightarrow{HG} + \overrightarrow{GF} = \overrightarrow{HG} - \overrightarrow{FG} = \begin{pmatrix} 4 \\ 14 \end{pmatrix} - \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}.$$

Step 2: magnitude using Pythagoras. **[M1 (independent; for a correct magnitude calculation)]**

$$|\overrightarrow{HF}| = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225}.$$

$$|\overrightarrow{HF}| = \boxed{15}.$$

[A1 (from fully correct figures)]

Mathsaurus tip: Reverse the arrow, flip the sign

$\overrightarrow{GF} = -\overrightarrow{FG}$. Whenever you need a vector that runs the "wrong way" compared to one you've been given, just put a minus sign in front. This is what lets you build a path between any two points using the vectors you have.

Notes from the examiner's report

Many students found this difficult and didn't know where to start. Several added the two given vectors and stopped, writing the resultant on the answer line as if that were the answer. Some reversed the x and y components for one of the vectors. A few got the correct vector $(9, 12)$ for $\overrightarrow{HF}$ but didn't take the magnitude. Only a small number followed through all the way to the correct answer of 15.

Total: M1 + M1 + A1 = 3 marks.

Question 18 Equation of a perpendicular line (4 marks)

The given line is $2x + y = 9$, which we rearrange to gradient-intercept form: **[M1 (for stating the gradient is -2 or rearranging correctly)]**

$$y = -2x + 9 \implies \text{gradient} = -2.$$

Step 1: gradient of the perpendicular. Perpendicular gradients multiply to -1: **[M1 ft (for a perpendicular gradient of $\frac{1}{2}$)]**

$$m_\perp = -\frac{1}{-2} = \frac{1}{2}.$$

Step 2: find the y-intercept c using point $(8, 11)$. The line has the form $y = \frac{1}{2}x + c$. Substitute $x = 8$, $y = 11$: **[M1 dep (for a correct method using the gradient and the point)]**

$$11 = \tfrac{1}{2}(8) + c \implies 11 = 4 + c \implies c = 7.$$

$$\boxed{y = \tfrac{1}{2}x + 7}.$$

[A1 oe]

Mathsaurus tip: Negative reciprocal

For perpendicular gradients, "flip and change the sign": $-2 \to \frac{1}{2}$. The product of the two gradients is -1, every time, for any pair of perpendicular non-vertical lines.

Notes from the examiner's report

Generally well done at this level. Students who lost marks usually didn't recognise the need to rearrange $2x + y = 9$ first, or used $m = 2$ or $m = -2$ as the perpendicular gradient (forgetting the negative reciprocal). Some scored M3 but rearranged incorrectly when finding c.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 19 Transformations of graphs (3 marks)

The minimum point of $y = \mathrm{f}(x)$ is at $(5, 4)$.

(i) $y = \mathrm{f}(x+5)$ shifts the graph 5 units to the *left*, so x goes from 5 to $5-5=0$. The y-coordinate is unchanged:

$$\boxed{(0,\ 4)}.$$

[B1]

(ii) $y = 3\mathrm{f}(x)$ stretches the graph vertically by a factor of 3. The x-coordinate is unchanged; the y-coordinate becomes $3 \times 4 = 12$:

$$\boxed{(5,\ 12)}.$$

[B1]

(iii) $y = \mathrm{f}(x) - 7$ shifts the graph 7 units *down*. The x-coordinate is unchanged; the y-coordinate becomes $4 - 7 = -3$:

$$\boxed{(5,\ -3)}.$$

[B1]

Mathsaurus tip: Inside vs outside the bracket

Anything *inside* the bracket (i.e. acting on x) affects the x-coordinate, and acts in the *opposite* direction to what it looks like: $\mathrm{f}(x+5)$ shifts left, $\mathrm{f}(x-5)$ shifts right. Anything *outside* the bracket (acting on the whole function) affects the y-coordinate and acts in the obvious direction.

Notes from the examiner's report

Students find graph transformations difficult and this question was no exception. Common wrong answers: $(5, 9)$ or $(10, 9)$ for part (i); $(15, 4)$ or $(15, 12)$ for part (ii); $(-2, 4)$ or $(-2, -3)$ for part (iii). The key is knowing which coordinate gets changed and in which direction.

Total: B1 + B1 + B1 = 3 marks.

Question 20 Quadratic inequality (3 marks)

Solve $10x^2 + 11x - 21 < 0$.

Step 1: factorise. We need two numbers that multiply to $10 \times (-21) = -210$ and add to 11. Try 21 and -10: $21 \times (-10) = -210$ and $21 + (-10) = 11$. **[M1 (for a correct method to solve the quadratic)]**

$$10x^2 + 11x - 21 = 10x^2 + 21x - 10x - 21 = x(10x + 21) - 1(10x + 21) = (10x + 21)(x - 1).$$

Step 2: critical values. **[A1 (dep on M1, for $x = 1$ and $x = -2.1$)]**

$$(10x + 21)(x - 1) = 0 \implies x = -\frac{21}{10} = -2.1 \text{ or } x = 1.$$

Step 3: pick the correct interval. $y = 10x^2 + 11x - 21$ is a positive quadratic (the coefficient of x^2 is positive), so its graph is U-shaped. It sits below the x-axis *between* its two roots, so the inequality < 0 is satisfied between them:

$$\boxed{-2.1 < x < 1}.$$

[A1 (dep on M1, oe)]

Mathsaurus tip: Sketch the parabola

For any quadratic inequality, sketch the parabola first. For a *positive* quadratic (positive coefficient of x^2), the graph is U-shaped: it sits below the x-axis between the roots and above outside. So quadratic < 0 means **between** the roots; quadratic > 0 means **outside** the roots. If it's a *negative* quadratic (negative coefficient of x^2, $\cap$-shaped), it's the other way around: the graph sits above the x-axis between the roots and below outside.

Notes from the examiner's report

Many students gained 2 marks for finding the critical values -2.1 and 1, but failed to write the inequality correctly at the end (e.g. giving $x < 1$ on its own, or $x > -2.1$ or $x < 1$ joined with "or"). Some worked backwards from a calculator result, writing the factorisation as $(x + 2.1)(x - 1)$, which doesn't expand to the given function and so scored zero (the question demanded clear algebraic working). A quick sketch always helps.

Total: M1 + A1 + A1 = 3 marks.

Question 21 Three-set Venn diagram (4 marks)

The Venn diagram has expressions in each region. We're told $n(A \cup B)' = 26$, that is, the total count of items *outside* both A and B is 26.

Step 1: form the equation. The regions outside both A and B are the parts of the diagram outside both those circles. From the diagram these are the $5x$ region (in C only) and the $3x + 2$ region (also outside A and B): **[M1 (for a correct equation in x)]**

$$5x + (3x + 2) = 26.$$

Solve: **[A1 (for $x = 3$)]**

$$8x + 2 = 26 \implies 8x = 24 \implies x = 3.$$

Step 2: find $n(A' \cap C)$. $A' \cap C$ is everything in C that is not in A. From the diagram, the regions in C but outside A are $5x$ (in C only) and $2x$ (in $B \cap C$ but not in A). **[M1 ft (for using their positive value of x in $7x$, i.e. correct regions for the set required)]**

$$n(A' \cap C) = 5x + 2x = 7x.$$

Step 3: substitute $x = 3$.

$$n(A' \cap C) = 7 \times 3 = \boxed{21}.$$

[A1 cao]

Mathsaurus tip: Shade before you count

Three-set Venn diagrams have seven distinct regions inside the circles, plus one outside. Shade the region you want before you start adding expressions. "In C but not in A" means inside the C circle and outside the A circle, including parts that are also in B.

Notes from the examiner's report

Many full-mark responses, with 2 marks a common partial score for students who solved the equation but couldn't pin down the right regions. A common slip was forming the equation $3x + 2 = 26$ (using only one region) instead of $5x + (3x + 2) = 26$. Some included $4x$ and $3x + 2$ wrongly in the final region. Very few used shading to keep track of which parts they wanted; this question is a great example of why shading helps.

Total: M1 + A1 + M1 + A1 = 4 marks.

Question 22 Simultaneous equations, one quadratic (5 marks)

Solve $x^2 + y^2 + y = 3$ and $x + 2 = y$.

Step 1: substitute the linear equation into the quadratic. **[M1 (substitution; allow one sign error)]**

$$x^2 + (x+2)^2 + (x+2) = 3.$$

Step 2: expand and tidy. **[M1 (dep on M1; simplified to a 3-term quadratic)]**

$$x^2 + x^2 + 4x + 4 + x + 2 = 3 \quad \Longrightarrow \quad 2x^2 + 5x + 6 = 3.$$

$$2x^2 + 5x + 3 = 0.$$

Step 3: factorise. Need two numbers multiplying to $2 \times 3 = 6$ summing to 5: 3 and 2. **[M1 ft (dep on M1; for solving their 3-term quadratic)]**

$$2x^2 + 5x + 3 = (2x+3)(x+1) = 0.$$

$$x = -\tfrac{3}{2} \text{ or } x = -1.$$

Step 4: find each y. $y = x + 2$: **[M1 (dep; for a correct method to find both other values)]**

$$x = -\tfrac{3}{2} \Rightarrow y = -\tfrac{3}{2} + 2 = \tfrac{1}{2}; \qquad x = -1 \Rightarrow y = -1 + 2 = 1.$$

$$\boxed{x = -\tfrac{3}{2},\ y = \tfrac{1}{2} \quad \text{or} \quad x = -1,\ y = 1}.$$

[A1 (dep on M2; for all four values)]

Mathsaurus tip: Substitute the easy one

The trick is always: rearrange the linear equation, then substitute it into the quadratic. Trying to "square" the linear equation (e.g. $(x+2)^2 = y^2$) and then eliminate y^2 is more work and a frequent source of errors.

Notes from the examiner's report

A good source of marks for students who'd revised this topic. Some had no idea where to start, or tried to "square" the linear equation. Those who reached the quadratic and could factorise usually went on to get full marks. A regular issue with the quadratic formula was writing 2 as the denominator instead of $2 \times a = 4$. Treating the 3 on the right as 0 when solving was another slip.

Total: M1 + M1 + M1 + M1 + A1 = 5 marks.

Question 23 Algebraic fractions, factor and cancel (4 marks)

Show that $\dfrac{16x^2-36}{x-7} \div \dfrac{2x^2+7x+6}{x^2-5x-14} - (7+8x) = n$.

Step 1: factorise everything in sight. **[M1 (independent; correct factorisation of $16x^2-36$)]**

$$16x^2-36 = 4(4x^2-9) = 4(2x-3)(2x+3).$$

[M1 (independent; correct factorisation of the other two quadratics)]

$$2x^2+7x+6 = (2x+3)(x+2), \qquad x^2-5x-14 = (x-7)(x+2).$$

Step 2: turn the division into multiplication by the reciprocal.

$$\frac{4(2x-3)(2x+3)}{x-7} \div \frac{(2x+3)(x+2)}{(x-7)(x+2)} = \frac{4(2x-3)(2x+3)}{x-7} \times \frac{(x-7)(x+2)}{(2x+3)(x+2)}.$$

Step 3: cancel. $(2x+3)$, $(x-7)$ and $(x+2)$ all cancel:

$$= 4(2x-3) = 8x-12.$$

Step 4: subtract $(7+8x)$. **[M1 (for a linear expression that should give the correct value of n)]**

$$8x-12-(7+8x) = 8x-12-7-8x = -19.$$

$$\boxed{n=-19}.$$

[A1 (dep on M2)]

Mathsaurus tip: Difference of two squares first

$16x^2-36$ looks scary but it's just a difference of two squares (after pulling out a 4): $4(2x-3)(2x+3)$. Spotting the factor of 4 is what makes the cancellation work, because $(2x+3)$ then matches a factor in the second numerator.

Notes from the examiner's report

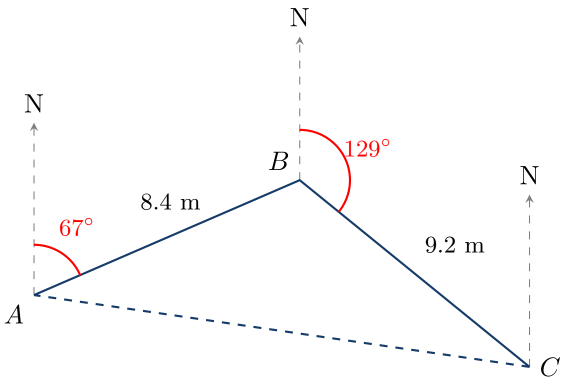
A demanding question. Students who didn't factorise often expanded into very long quartic expressions that rarely came out right. A frustrating slip: reaching $8x-12-(7+8x)$ and then writing $8x-12-7+8x$ (sign error on the bracket), giving $n=16x-19$ instead of -19. $16x^2-36$ was frequently factorised as $(2x-3)(2x+3)$, losing the factor of 4. Some "derived" a value of x from the equation and substituted, which is not a valid approach.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 24 Bearings with the cosine and sine rules (6 marks)

$AB = 8.4$ m, $BC = 9.2$ m, B is on bearing 067° from A, C is on bearing 129° from B. Find the bearing of A from C.

Sketch the situation first. The question gives no diagram, so draw your own. Mark A, then B at bearing 067° from A, then C at bearing 129° from B, and add a north-line at each point.



Step 1: find $\angle ABC$. The two north-lines (at A and B) are parallel and AB cuts across both. By co-interior angles, the angle between BA and the north-line at B, measured *anti-clockwise* from the north-line, is:

$$180° - 67° = 113°.$$

The bearing of C from B is 129°, the angle from B's north going *clockwise* to BC. Going all the way around point B is 360°, so:

$$\angle ABC = 360° - 129° - 113° = 118°.$$

[M1 (a diagram showing 118°, or use of 118°)]

Step 2: cosine rule for AC. **[M1 (for a correct method to find AC^2)]**

$$AC^2 = 8.4^2 + 9.2^2 - 2(8.4)(9.2)\cos 118° = 70.56 + 84.64 - 154.56 \cos 118° \approx 227.76.$$

[M1 (for a correct method to find AC)]

$$AC \approx \sqrt{227.76} \approx 15.09 \text{ m}.$$

Step 3: sine rule for $\angle ACB$. **[M1 (dep; for a correct sine-rule statement for $\angle ACB$ or $\angle BAC$)]**

$$\frac{\sin(\angle ACB)}{AB} = \frac{\sin(\angle ABC)}{AC} \implies \frac{\sin(\angle ACB)}{8.4} = \frac{\sin 118°}{15.09}.$$

[M1 (a completely correct statement leading to the angle)]

$$\angle ACB = \sin^{-1}\left(\frac{8.4 \sin 118°}{15.09}\right) \approx 29.43°.$$

Step 4: bearing of A from C. At C, the north-line is parallel to the one at B and BC cuts across both. The bearing of C from B is 129°, so by co-interior angles, the angle between CB and the north-line at C, measured *anti-clockwise* from the north-line, is:

$$180° - 129° = 51°.$$

The line CA sits a further $\angle ACB = 29.43°$ anti-clockwise from CB. So CA is $51° + 29.43° = 80.43°$ anti-clockwise from C's north-line.

A bearing is measured *clockwise* from north, so:

$$\text{bearing of } A \text{ from } C = 360° - 51° - 29.43° = 279.57°.$$

To the nearest degree:

$$\boxed{280°} \quad \text{(accept } 279° - 280°\text{)}.$$

[A1 cao]

Mathsaurus tip: Mark every angle on the sketch

With no diagram given, half the work is on the page in front of you before you do any trig. Once $\angle ABC = 118°$ is on your sketch, the cosine rule and sine rule are routine; the trickiest mark is usually the final step, where co-interior angles between the parallel north-lines at B and C give you the angle from CB to C's north-line directly.

Notes from the examiner's report

Set at the highest level for the paper, no diagram given as scaffolding. Students who drew an annotated diagram and worked out $\angle ABC = 118°$ usually picked up the first mark and then several more. A few used a scale drawing (rarely fully accurate; 278° was a common scale-drawing answer). Some pleasing solutions; 5 marks (everything except the final bearing direction) was common, and full 6-mark scripts were seen at the expected rate.

Total: M1 + M1 + M1 + M1 + M1 + A1 = 6 marks.

Question 25 Equilateral triangle with inscribed circle (5 marks)

Equilateral triangle ABC has an inscribed circle of radius x (centre O). The shaded area is the part of the triangle outside the circle, and equals nx^2.

Step 1: side length of the triangle. Drop a perpendicular from O to AB; this has length x (the radius) and meets AB at its midpoint. Look at the right triangle from O to that midpoint to A. The angle at A is half of $60°$, so $30°$. Hence: **[M1 (for a correct trig setup)]**

$$\tan 30° = \frac{x}{\frac{1}{2}AB} \implies \frac{1}{2}AB = \frac{x}{\tan 30°}.$$

So $AB = \dfrac{2x}{\tan 30°}$. Using $\tan 30° = \dfrac{1}{\sqrt{3}}$:

$$AB = 2x\sqrt{3} = 2\sqrt{3}\,x.$$

[M1 (for an expression for the side of the triangle)]

Step 2: area of the triangle. Use Area $= \frac{1}{2}ab\sin C$ with $a = b = AB$ and $C = 60°$: **[M1 (for a correct expression for the area of triangle ABC)]**

$$\text{Area}(\triangle ABC) = \tfrac{1}{2}(2\sqrt{3}\,x)(2\sqrt{3}\,x)\sin 60° = \tfrac{1}{2} \times 12x^2 \times \tfrac{\sqrt{3}}{2} = 3\sqrt{3}\,x^2.$$

Step 3: shaded area. Subtract the inscribed circle's area, πx^2: **[M1 (for a correct expression for the shaded area)]**

$$\text{Shaded} = 3\sqrt{3}\,x^2 - \pi x^2 = (3\sqrt{3} - \pi)\,x^2.$$

So $n = 3\sqrt{3} - \pi$. Evaluate:

$$n = 5.196\ldots - 3.1416\ldots = 2.054\ldots$$

$$\boxed{n = 2.05 \text{ (to 3 s.f.)}} \quad \text{(accept 2.05 – 2.06).}$$

[A1 cao]

Mathsaurus tip: Tie everything to x

The trap on this question is adding a new variable for the side length, then trying to find x separately. There's no need: the radius x determines the whole figure, and the side length comes out as $2\sqrt{3}\,x$. Keep everything in terms of x from the start and the x^2 pulls out cleanly at the end.

Notes from the examiner's report

A challenging question with multiple valid approaches. Many made no attempt or just wrote formulae. A common mistake: assuming the side of the triangle was x or $2x$, with no justification. Some students tried to "find" x, not realising it was a parameter that should stay in the answer. The successful approach was usually $\frac{1}{2}ab\sin C$ with $C = 60°$ and $a = b = AB$; once AB was expressed in terms of x (via trigonometry on the half-triangle), the rest fell into place.

Total: M1 + M1 + M1 + M1 + A1 = 5 marks.

End of paper. Total: 100 marks.