MATHSAURUS

Pearson Edexcel International GCSE Mathematics A (4MA1)

November 2024 Paper 1H

Higher Tier · Calculator permitted

Mark schemes you can actually read.

Students always tell me the official mark schemes are hard to read. That's because those documents are written for examiners and teachers, not for the people actually sitting the exam. So I've fixed that. I've taken the official mark scheme, the worked solutions, and the most useful parts of the examiner's report, and turned them into one document written for you. Every step is explained, every mark is labelled, and I've highlighted what real students got wrong on this paper, so you don't have to read the full report yourself to learn from it. I really hope it helps you with your studies. If it does, there's a lot more at mathsaurus.com: courses, video tutorials, and resources I've spent years building up. Have a look once you're done.

Dr Kevin Olding, creator of mathsaurus.com

Question index *(click the links to jump to that question)*

How to read this document

These worked solutions are designed as an *expanded mark scheme*. We show every step of the working, explain the reasoning, and indicate exactly where each mark is awarded. There are three things to look out for as you read.

Mark annotations

Inline labels like **[M1]**, **[A1]** and **[B1]** appear next to the working step where each mark is earned. These follow the official Edexcel mark-scheme notation:

- **[M1]**, **[M2]**, ... **Method marks.** Awarded for using a correct method, even if the arithmetic later goes wrong. You earn these by writing down the right approach.
- **[A1]**, **[A2]**, ... **Accuracy marks.** Awarded for the correct value or expression. An A mark almost always depends on having earned the related M mark first.
- **[B1]**, **[B2]**, ... **Unconditional accuracy marks.** Awarded for a correct answer that doesn't depend on any method mark (e.g. writing down a single value).

You will also see these abbreviations:

- **cao**, *correct answer only.* No follow-through allowed: the value must be exactly right.
- **ft**, *follow through.* A method applied correctly to your own earlier (possibly wrong) values can still earn the mark.
- **oe**, *or equivalent.* The answer can take any equivalent form (e.g. $\frac{7}{5}$ or 1.4 or $1\frac{2}{5}$).
- **dep**, *dependent.* You can only earn this mark if you have earned an earlier specified mark.
- **isw**, *ignore subsequent working.* Once you have reached the answer, harmless extra working will not lose you marks.

Mathsaurus tips (blue boxes)

Blue boxes give you the *why*: the strategy behind a method, an insight that makes a hard step easy, or a common trap and how to avoid it. Each tip carries its own short label so you can scan for the one you need.

Notes from the examiner's report (yellow boxes)

Yellow boxes flag what real students got wrong on each question, drawn from the official Edexcel examiner's report. We give a brief, paraphrased note here rather than the full report; the complete examiner's commentary for this paper is available on the Pearson qualifications website.

Question 1 Estimating the mean from a grouped frequency table (5 marks)

(a) Modal class. The modal class is simply the class with the highest frequency. Looking at the table, the frequency 18 is the largest, so:

$$\boxed{10 < p \leq 15} \quad \textbf{[B1]}$$

(b) Estimate of the mean. Because we only know which class each worker fell into, not their exact pay, we use the **midpoint** of each class as the best single estimate.

Mathsaurus tip: Why we use midpoints

For a class like $10 < p \leq 15$, every value lies somewhere between 10 and 15. With no further information, the midpoint 12.5 is the fairest single estimate.

Midpoints: 12.5, 17.5, 22.5, 27.5, 32.5.

Multiply each midpoint by its frequency and sum: **[M2 (or M1 for at least 4 correct products)]**

$$12.5 \times 18 + 17.5 \times 16 + 22.5 \times 14 + 27.5 \times 8 + 32.5 \times 4$$

$$= 225 + 280 + 315 + 220 + 130 = 1170.$$

Now divide by the total frequency (60): **[M1 dep]**

$$\bar{p} \approx \frac{1170}{60} = \boxed{19.5 \text{ dollars}}.$$

[A1]

Notes from the examiner's report

A common slip in part (b) was dividing 1170 by the class width 5, or by the total of the candidate's own (incorrect) frequency column instead of using the given total of 60. Make sure you divide by the total frequency.

Total: B1 + M2 + M1 + A1 = 5 marks.

Question 2 Constructing the bisector of angle ABC (2 marks)

This is a ruler-and-compass construction. Without a hand-drawn figure, here is the precise method:

1. Place the compass point on B. Draw an arc that crosses both BA and BC. Call the crossing points X and Y.
2. Without changing the radius, place the compass on X and draw an arc inside the angle.
3. Keeping the same radius, place the compass on Y and draw another arc that crosses the previous one. Call the crossing point Z.
4. Draw the straight line from B through Z. This is the bisector. **[B2 (B1 for all arcs but no bisector, or correct bisector with no arcs)]**

Mathsaurus tip: Why the construction works

X and Y are the same distance from B, and Z is the same distance from both X and Y. So B and Z both lie on the perpendicular bisector of XY, and that line bisects the angle at B.

Mark scheme tip: you must leave *all the construction arcs* on the page. Erasing them costs marks.

Notes from the examiner's report

The most common slip was drawing arcs centred at A and C rather than from points equidistant from B. Some students drew freehand without using compasses, which gained no marks. Some simply drew a straight line from B to a point inside the angle with no arcs at all.

Total: B2 = 2 marks.

Question 3 Index laws, expanding, solving (6 marks)

(a) $(p^3)^5$.

Using $(a^m)^n = a^{mn}$:

$$(p^3)^5 = p^{3\times 5} = \boxed{p^{15}}.$$

[B1 cao]

(b) Expand and simplify $2n(4n+3) + n(n-4)$.

Multiply each bracket out, term by term: **[M1 (for an expansion with at least 3 of 4 correct terms)]**

$$2n(4n+3) = 8n^2 + 6n, \qquad n(n-4) = n^2 - 4n.$$

Now add and collect like terms:

$$8n^2 + 6n + n^2 - 4n = \boxed{9n^2 + 2n}.$$

[A1 oe]

(c) Solve $\dfrac{2x+5}{3} = 4 - x$.

The fraction is the obstacle, so multiply both sides by 3 to remove it: **[M1 (for clearing the fraction)]**

$$2x + 5 = 3(4-x) = 12 - 3x.$$

Now collect x terms on one side, numbers on the other: **[M1 ft (for a correct rearrangement)]**

$$2x + 3x = 12 - 5 \quad\Longrightarrow\quad 5x = 7 \quad\Longrightarrow\quad \boxed{x = \tfrac{7}{5}}.$$

[A1 oe]

Notes from the examiner's report

(b) Some students wrote $2n(4n+3) = 6n^2 + 6n$ instead of $8n^2 + 6n$. The 2 must multiply both terms inside the bracket. (c) Most students did well; a frequent slip was clearing the fraction without multiplying *both* terms on the right-hand side by 3.

Total: B1 + M1+A1 + M1+M1+A1 = 6 marks.

Question 4 Venn diagram (3 marks)

The diagram shows: only in A: $\{9\}$. In both A and B: $\{3, 7\}$. Only in B: $\{5, 2\}$. Outside both: $\{4, 6, 8\}$.

(a) Set B contains everything inside the B circle (including the overlap):

$$B = \boxed{2,\ 3,\ 5,\ 7}.$$

[B1]

(b) $A \cap B$ is the overlap only:

$$A \cap B = \boxed{3,\ 7}.$$

[B1]

(c) A' (read "A prime" or "not A") is everything in the universal set $\mathcal{E}$ that is *not* in A:

$$A' = \boxed{2,\ 4,\ 5,\ 6,\ 8}.$$

[B1]

Mathsaurus tip: Don't forget the universal set

A common slip is to forget the elements outside both circles. They are still in the universal set, so they belong in A'.

Notes from the examiner's report

All parts were generally answered well. Marks were lost most often when students missed an element (especially in A') or duplicated one. Writing the elements neatly in any order, separated by commas or spaces, is fine.

Total: B1 + B1 + B1 = 3 marks.

Question 5 Volume of a cylinder, in litres (4 marks)

Radius $r = 70$ cm, height $h = 18$ cm. Use $V = \pi r^2 h$: **[M1 (for use of $\pi r^2 h$)]**

$$V = \pi \times 70^2 \times 18 = 88\,200\pi \text{ cm}^3 \approx 277\,088.47 \text{ cm}^3.$$

[A1]

To convert to litres, recall 1 litre = 1000 cm^3: **[M1 (for dividing the volume by 1000)]**

$$V \approx \frac{277\,088.47}{1000} \approx 277.088\ldots \text{ litres.}$$

To the nearest litre:

$$\boxed{277 \text{ litres}}.$$

[A1 awrt 277]

Mathsaurus tip: Don't forget the unit conversion

The conversion to litres is the step that catches people out. Once you have the volume in cm^3, the key fact is 1 cm^3 = 1 ml, so 1000 cm^3 = 1 litre. If you'd worked in metres from the start (radius 0.7 m, height 0.18 m), you'd use 1 m^3 = 1000 litres instead.

Notes from the examiner's report

The first two marks were straightforward for most students. The conversion to litres was the sticking point: many students divided by 100 or 10 000 instead of 1000, or simply did not know that 1 cm^3 = 1 ml.

Total: M1 + A1 + M1 + A1 = 4 marks.

Question 6 Highest common factor from prime factorisations (2 marks)

$$A = 2^3 \times 5^4 \times 7 \times 11, \quad B = 2^2 \times 5^2 \times 7^2, \quad C = 2^2 \times 5^3 \times 7^4.$$

For the HCF, take each prime that appears in **all three** numbers, raised to the **lowest** power.

- Prime 2: powers are $3, 2, 2$. Lowest is 2.
- Prime 5: powers are $4, 2, 3$. Lowest is 2.
- Prime 7: powers are $1, 2, 4$. Lowest is 1.
- Prime 11: only appears in A, so we do not include it.

$$\text{HCF} = \boxed{2^2 \times 5^2 \times 7} \quad (= 700).$$

[B2 (B1 for two of the three primes correct, or for 700)]

Notes from the examiner's report

Many students confused HCF with LCM and took the highest powers instead of the lowest. Some incorrectly included the 11 (which appears only in A). Note that just writing 700 was awarded only 1 mark; the question asks for the answer as a product of prime factors, so the $2^2 \times 5^2 \times 7$ form is required for full marks.

Total: B2 = 2 marks.

Question 7 Comparing percentage discounts (4 marks)

Shop A: 16% off £475: **[M1 (for shop A's discount)]**

$$0.16 \times 475 = £76 \text{ off.}$$

Shop B: The customer pays £408 *after* a 15% discount, so £408 represents 85% of the normal price: **[M1 (for setting up the reverse percentage)]**

$$\text{Normal price} = \frac{408}{0.85} = £480.$$

[M1 (for completing the reverse percentage)] The discount is therefore $480 - 408 = £72$ off.

Comparing: $£76 > £72$, so:

$$\boxed{\text{Shop A gives more money off } (£76 \text{ vs } £72).}$$

[A1 dep on M2 (with both 72 and 76 seen)]

Mathsaurus tip: Don't compare percentages directly

The trap here is comparing percentages directly ($16\% > 15\%$, "so A wins"). That is not a valid argument because the normal prices differ. You must convert each to an actual amount of money saved.

Notes from the examiner's report

Read the question, then read your answer. A surprising number of students worked out 76 and 72 correctly, then never said which shop wins. The mark is for naming the shop, not just the arithmetic. The most common method error was using 1.15 as a multiplier (i.e. *adding* 15% to 408) instead of recognising that 408 is 85% of the normal price.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 8 Quadratic factorising and an inequality (6 marks)

(a)(i) Factorise $x^2 + 5x - 24$.

We need two numbers that **multiply to** -24 and **add to** $+5$. Trying pairs of factors of 24: $(8, -3)$ works since $8 \times (-3) = -24$ and $8 + (-3) = 5$. **[M1 (for any pair multiplying to -24 with sum 5)]**

$$x^2 + 5x - 24 = \boxed{(x+8)(x-3)}.$$

[A1]

(a)(ii) Hence solve $x^2 + 5x - 24 = 0$.

The factorised form is zero exactly when one of the brackets is zero:

$$x + 8 = 0 \text{ or } x - 3 = 0 \quad \Longrightarrow \quad \boxed{x = -8 \text{ or } x = 3}.$$

[B1 ft]

(b) Solve $3y + 5 > 7y - 10$.

Treat the inequality just like an equation, but stay alert: *multiplying or dividing by a negative number flips the sign.* Here we will avoid that by collecting y terms on the side that gives a positive coefficient. **[M1 (for a correct rearrangement; $=$ condoned)]**

$$3y + 5 > 7y - 10 \quad \Longrightarrow \quad 5 + 10 > 7y - 3y \quad \Longrightarrow \quad 15 > 4y.$$

[M1 (for collecting; sign condoned)]

$$\frac{15}{4} > y \quad \Longrightarrow \quad \boxed{y < \tfrac{15}{4}} \quad (\text{or } y < 3.75).$$

[A1 (must have correct inequality sign)]

Notes from the examiner's report

The negative coefficient on y tripped many students up. Common wrong final answers were $y = 3.75$ (no inequality sign), $y > 3.75$ (sign reversed), or just 3.75 on the answer line. In part (a)(i) the most common slip was the wrong sign pattern, e.g. writing $(x-8)(x+3)$. Always multiply back out as a check.

Total: M1+A1 + B1 + M1+M1+A1 = 6 marks.

Question 9 Standard form (3 marks)

(a) Write 8.4×10^{-5} as an ordinary number.

The exponent -5 means move the decimal point 5 places to the left:

$$8.4 \times 10^{-5} = \boxed{0.000\,084}.$$

[B1 cao]

(b) Work out $(6.5 \times 10^{-40}) \times (8 \times 10^{185})$.

Multiply the number parts and add the powers: **[M1 (for 52×10^{145} or 5.2×10^{n})]**

$$(6.5 \times 8) \times 10^{-40+185} = 52 \times 10^{145}.$$

This is *not yet* in standard form because $52 \geq 10$. Adjust:

$$52 \times 10^{145} = 5.2 \times 10^{1} \times 10^{145} = \boxed{5.2 \times 10^{146}}.$$

[A1]

Mathsaurus tip: Adjusting back to standard form

Standard form requires $1 \leq a < 10$. After multiplying, 52 falls outside that range. Pulling out a factor of 10^1 shifts the exponent up by 1, giving 5.2×10^{146}.

Notes from the examiner's report

Common wrong answers in (b): 5.2×10^{144} (forgetting that $52 = 5.2 \times 10^1$ *adds* 1 to the exponent), 5.2^{146} (dropping the $\times 10$ entirely), or 5.2×10^{-146} (sign error on the exponent).

Total: B1 + M1 + A1 = 3 marks.

Question 10 Similar right-angled triangles (5 marks)

We are told the two triangles are similar, so corresponding sides are in the same ratio. Reading off the matching sides from the marked angles:

$$AB \leftrightarrow DE, \quad AC \leftrightarrow DF, \quad BC \leftrightarrow EF.$$

Step 1: find the missing side DE in the larger triangle. Use Pythagoras on triangle DEF: **[M1 (for applying Pythagoras correctly)]**

$$DE^2 + 24^2 = 51^2 \implies DE^2 = 2601 - 576 = 2025 \implies DE = 45 \text{ cm}.$$

[M1 (for square rooting)]

Step 2: find the scale factor. AB corresponds to DE: **[M1 (for a correct method to find the scale factor)]**

$$\text{scale factor (small to large)} = \frac{DE}{AB} = \frac{45}{7.5} = 6.$$

Step 3: find x. AC corresponds to $DF = 24$. Since the larger triangle is 6 times bigger: **[M1 dep (for using the scale factor with 24)]**

$$x = \frac{DF}{6} = \frac{24}{6} = \boxed{4 \text{ cm}}.$$

[A1 dep on M2]

Mathsaurus tip: Alternative: trigonometry

Because the triangles are similar, the marked angles match. Find the angle from the larger triangle and reuse it in the smaller one:

$$\cos(\text{angle at } F) = \frac{24}{51} \implies \text{angle} \approx 61.93^\circ.$$

The same angle sits at C in the smaller triangle, where $AB = 7.5$ is opposite and $AC = x$ is adjacent:

$$\tan 61.93^\circ = \frac{7.5}{x} \implies x = \frac{7.5}{\tan 61.93^\circ} = 4 \text{ cm}.$$

This is also fully credited by the mark scheme.

Notes from the examiner's report

Some students wrote $51^2 + 24^2$ for Pythagoras instead of $51^2 - 24^2$. The longest side here is 51, so its square equals the sum of the other two squares, not the other way round. Trigonometry was an equally acceptable alternative.

Total: M1 + M1 + M1 + M1 + A1 = 5 marks.

Question 11 Plotting $y = 2(x + 1/x)$ (4 marks)

(a) Substitute each x value into $y = 2\left(x + \frac{1}{x}\right)$. **[B2 (B1 for two or three correct y-values)]**

x	0.5	1	2	3	4	5	6
y	**5**	**4**	5	6.7	**8.5**	**10.4**	12.3

Sample working: at $x = 0.5$: $y = 2(0.5 + 2) = 5$. At $x = 4$: $y = 2(4 + 0.25) = 8.5$. At $x = 5$: $y = 2(5 + 0.2) = 10.4$.

(b) Plot the seven points carefully on the grid and join them with a *smooth curve*, not straight line segments. **[M1 ft dep (for plotting at least 6 points correctly)]** **[A1 (correct curve)]**

Mathsaurus tip: Sketch-checking your table

Before plotting, glance down your y-values: $5, 4, 5, 6.7, 8.5, 10.4, 12.3$. The values dip then rise, with 4 as the lowest. If a value broke that pattern (e.g. if you had written $y = 6$ at $x = 1$), it is worth re-checking the calculation before plotting. Smooth curves do not have sudden zig-zags.

Notes from the examiner's report

Part (a)'s table was almost always completed correctly. In part (b), some students used a ruler to draw straight lines between points (particularly between $(0.5, 5)$, $(1, 4)$ and $(2, 5)$) instead of a smooth curve, which loses the accuracy mark.

Total: B2 + M1 + A1 = 4 marks.

Question 12 Two-step trigonometry (4 marks)

We have a right-angled triangle ABC with the right angle at B. Point D lies on BC. Given: $AB = 4250$ m, angle $BAD = 47°$, angle $BCA = 24°$.

We want DC. Since D lies on BC, we have $DC = BC - BD$. So find each piece using one of the right-angled sub-triangles.

Step 1: find BD. In triangle ABD (right-angled at B), the side BD is *opposite* the 47° angle and AB is *adjacent*. So: **[M1 (for a correct trig statement in either sub-triangle)]**

$$\tan 47° = \frac{BD}{4250} \implies BD = 4250 \tan 47° \approx 4557.567 \text{ m}.$$

[M1 (for one length found)]

Step 2: find BC. In triangle ABC (right-angled at B), $AB = 4250$ is *opposite* the 24° angle and BC is *adjacent*: **[M1 (for a complete method to find DC)]**

$$\tan 24° = \frac{4250}{BC} \implies BC = \frac{4250}{\tan 24°} \approx 9545.656 \text{ m}.$$

Step 3: subtract.

$$DC = BC - BD \approx 9545.656 - 4557.567 \approx \boxed{4988 \text{ m}}.$$

[A1 (allow 4932 – 4990)]

Mathsaurus tip: Use the right sub-triangle

Whenever a question gives several angles and asks for a length, ask: "which sub-triangle has all the information I need?" Treating ABD and ABC separately (each a clean right-angled triangle) is far easier than trying to use the sine or cosine rule on the whole shape.

Notes from the examiner's report

Many students gained full marks. The most common slip was rearranging tan incorrectly, e.g. writing $BD = 4250/\tan 47°$ rather than $BD = 4250 \tan 47°$. Be careful which side is opposite and which is adjacent before dividing.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 13 Tree diagram for two consecutive days (4 marks)

(a) Filling in the missing probabilities.

The pair on each "branching" must add to 1: **[B2 (B1 for one correct pair)]**

- Saturday: $P(\text{not early}) = 1 - 0.7 = \boxed{0.3}$.
- Sunday given Saturday early: $P(\text{not early}) = 1 - 0.9 = \boxed{0.1}$.
- Sunday given Saturday not early: $P(\text{early}) = 0.6$, $P(\text{not early}) = 1 - 0.6 = \boxed{0.4}$.

(b) Probability of early on *both* days.

Travel along the (early, early) branch and multiply: **[M1 ft (for 0.7×0.9)]**

$$P(\text{both early}) = 0.7 \times 0.9 = \boxed{0.63}.$$

[A1 ft oe]

Mathsaurus tip: AND means multiply, OR means add

On a tree diagram, travelling along a single branch is an AND event (Saturday early AND Sunday early): multiply the probabilities. Combining outcomes from different branches is an OR event: add the (already multiplied) branch probabilities.

Notes from the examiner's report

Most students got (a) right. A few wrote integers instead of decimals on the branches. In (b), the standard slip is *adding* along the branch instead of multiplying, which gives an answer greater than 1 (a quick way to spot the mistake).

Total: B2 + M1 + A1 = 4 marks.

Question 14 Inverse proportion (3 marks)

"B is inversely proportional to the square of d" means **[M1 (constant of proportionality must be a symbol)]**

$$B = \frac{k}{d^2} \qquad \text{for some constant } k.$$

Substitute the known values $B = 0.25$, $d = 12$: **[M1 (for substitution into a correct formula)]**

$$0.25 = \frac{k}{12^2} = \frac{k}{144} \quad \Longrightarrow \quad k = 0.25 \times 144 = 36.$$

So:

$$\boxed{B = \frac{36}{d^2}}.$$

[A1 oe]

Mathsaurus tip: $\propto$ vs $=$

A common error: writing $B = \frac{1}{d^2}$ at the start with no constant. The proportionality *symbol* $\propto$ becomes an *equals sign* only when you introduce a constant of proportionality.

Notes from the examiner's report

Two slips to watch for: (1) writing $B = kd^2$, which is direct proportion and gets you no marks here; (2) substituting $d = 12$ without squaring it. Read the question, then check the shape of your formula before you plug numbers in.

Total: M1 + M1 + A1 = 3 marks.

Question 15 Expanding three brackets (3 marks)

Write $3x(2x-1)(5x+4)$ in the form ax^3+bx^2+cx.

Multiply two brackets at a time. Easiest is to combine the second and third first: **[M1 (for an expansion of any pair with at most one error)]**

$$(2x-1)(5x+4) = 10x^2+8x-5x-4 = 10x^2+3x-4.$$

Now multiply by $3x$: **[M1 ft dep (for the second multiplication; one error allowed)]**

$$3x(10x^2+3x-4) = \boxed{30x^3+9x^2-12x}.$$

[A1 cao]

So $a=30$, $b=9$, $c=-12$.

Mathsaurus tip: Two brackets at a time

Three brackets are easier to handle if you collapse two into one first, then multiply by the third. Trying to do all three in your head usually loses a sign somewhere.

Notes from the examiner's report

Most students gained full marks. A few multiplied two of the brackets, then *a different* two, and tried to combine the results, which leads to a tangle. Stick to one bracket pair at a time. Some students who got the correct answer then attempted to factorise; if done correctly $3x(10x^2+3x-4)$ still scored full marks, but if done incorrectly only 2 marks were awarded.

Total: M1 + M1 + A1 = 3 marks.

Question 16 Sector area from triangle area (4 marks)

The sector $OAPB$ has angle $50°$ at the centre. We are told the area of triangle OAB is 120 cm^2. The two equal sides of the triangle (since OA and OB are radii) both have length r.

Step 1: find r^2 using the area-of-a-triangle formula. **[M1 (for setting up $\frac{1}{2}r^2 \sin 50 = 120$ or equivalent)]**

$$\text{Area} = \tfrac{1}{2}ab\sin C \quad \Longrightarrow \quad 120 = \tfrac{1}{2} \times r \times r \times \sin 50°.$$

$$r^2 = \frac{2 \times 120}{\sin 50°} = \frac{240}{\sin 50°} \approx 313.298.$$

[M1 (for rearranging to r^2)]

Step 2: use the sector-area formula. A full circle has area πr^2, and a sector is the fraction $\frac{\theta}{360}$ of that: **[M1 (for the sector area formula with r^2 and $50/360$)]**

$$\text{Sector area} = \frac{50}{360} \times \pi \times r^2 = \frac{50}{360} \times \pi \times 313.298 \approx 136.78.$$

To 3 sf:

$$\boxed{137 \text{ cm}^2}.$$

[A1 awrt 137]

Mathsaurus tip: Spot the r^2 shortcut

You don't actually need to find r on its own, only r^2. Spotting that the sector formula uses r^2 (the same quantity the triangle formula gave you) saves a square-root step.

Notes from the examiner's report

Pleasingly, many students gave fully correct solutions. The most common error worth flagging: some students worked out the sector area correctly as 137, then *subtracted* the given triangle area 120 to give 17, gaining only 3 of 4 marks. The question asks for the sector, not the segment.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 17 Surds (4 marks)

(a) Express $\sqrt{675}$ in the form $n\sqrt{27}$.

Look for a multiple of 27 inside 675:

$$675 = 25 \times 27 \implies \sqrt{675} = \sqrt{25 \times 27} = \sqrt{25} \times \sqrt{27} = \boxed{5\sqrt{27}}.$$

[B1 (allow $n = 5$ as long as $5\sqrt{27}$ appears in working)]

So $n = 5$.

(b) Show that $\frac{5 - \sqrt{2}}{\sqrt{2} - 1}$ can be written as $a + b\sqrt{2}$ for integers a, b.

The fraction has a surd in the denominator. *Rationalise* by multiplying top and bottom by the conjugate $\sqrt{2} + 1$: **[M1 (for multiplying by the conjugate)]**

$$\frac{5 - \sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}.$$

Numerator: **[M1 (numerator expanded to 4 correct terms)]**

$$(5 - \sqrt{2})(\sqrt{2} + 1) = 5\sqrt{2} + 5 - 2 - \sqrt{2} = 4\sqrt{2} + 3.$$

Denominator:

$$(\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1.$$

So:

$$\frac{5 - \sqrt{2}}{\sqrt{2} - 1} = \frac{4\sqrt{2} + 3}{1} = \boxed{3 + 4\sqrt{2}},$$

[A1 (or for stating $a = 3, b = 4$)] which is in the form $a + b\sqrt{2}$ with $a = 3$, $b = 4$ (both integers, as required).

Mathsaurus tip: Why the conjugate

The conjugate of $\sqrt{2} - 1$ is $\sqrt{2} + 1$. Multiplying them uses the difference of two squares: $(\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1$, which removes the surd from the denominator.

Notes from the examiner's report

(a) Just writing "5" alone wasn't credited; you must show $5\sqrt{27}$ in your working. (b) The standard slip was rationalising by $\sqrt{2} - 1$ (the same as the denominator) instead of the conjugate $\sqrt{2} + 1$. Some students entered the expression directly into a calculator and copied the screen. The question asks for working, so this scored zero.

Total: B1 + M1 + M1 + A1 = 4 marks.

Question 18 Reading a histogram (4 marks)

Recall that **frequency = frequency density × class width**.

Step 1: count the students between 20 and 60 minutes. **[M1 (for at least two correct sub-frequencies)]**

The bars covering that range are 20–25, 25–30, and 30–60:

$$\begin{aligned}
20\text{–}25 &: \quad 0.8 \times 5 = 4 \\
25\text{–}30 &: \quad 4 \times 5 = 20 \\
30\text{–}60 &: \quad 2.5 \times 30 = 75 \\
& \qquad\qquad\quad \overline{\ \ 99\ \ }
\end{aligned}$$

[M1 (for the value 99 in the 20–60 range)]

Step 2: count all the students in total. Add the other two bars:

$$\begin{aligned}
0\text{–}10 &: \quad 1.5 \times 10 = 15 \\
10\text{–}25 &: \quad 0.8 \times 15 = 12 \\
25\text{–}30 &: \quad 4 \times 5 = 20 \\
30\text{–}60 &: \quad 2.5 \times 30 = 75 \\
60\text{–}65 &: \quad 4.6 \times 5 = 23 \\
& \qquad\qquad\quad \overline{\ \ 145\ \ }
\end{aligned}$$

[M1 (for forming a complete fraction)]

Step 3: form the fraction.

$$\text{Fraction} = \boxed{\frac{99}{145}}.$$

[A1 (must be a fraction)]

Mathsaurus tip: Why frequency density

The class widths in this histogram are uneven (some bars are 5 minutes wide, others 10, 15, or 30). That is exactly why the y-axis is *frequency density* rather than *frequency*: the area of each bar gives the count.

Notes from the examiner's report

Most students reached 99 for the 20–60 minute range. The standard slip was forgetting that the answer needs to be a *fraction*: students need the total (145 here) too. A handful counted small or large squares on the grid, which is acceptable provided the working is clear.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 19 Arithmetic series with two pieces of information (5 marks)

Let the first term be a and the common difference be d.

Equation 1 (from $S_{30} = 4395$). Use $S_n = \frac{n}{2}\big[2a + (n-1)d\big]$: **[M1 (for using the sum formula)]**

$$\frac{30}{2}\big[2a + 29d\big] = 4395 \implies 15(2a + 29d) = 4395 \implies 2a + 29d = 293.$$

Equation 2 (from $u_{10} + u_{20} = 284$). Use $u_n = a + (n-1)d$: **[M1 (for using u_n to form a correct equation)]**

$$(a + 9d) + (a + 19d) = 284 \implies 2a + 28d = 284.$$

Subtracting: the $2a$ terms cancel: **[M1 dep on M2 (for a correct elimination step)]**

$$(2a + 29d) - (2a + 28d) = 293 - 284 \implies d = 9.$$

Then a: $2a + 28(9) = 284 \Rightarrow 2a = 32 \Rightarrow a = 16.$

Compute S_{45}: [M1 dep (for using S_n with $a = 16$, $d = 9$)]

$$S_{45} = \frac{45}{2}\big[2(16) + 44(9)\big] = \frac{45}{2}(32 + 396) = \frac{45}{2} \times 428 = 45 \times 214 = \boxed{9630}.$$

[A1 dep]

Mathsaurus tip: Layout matters

This is a multi-step problem with two unknowns. Students who line their work up clearly (one equation per line, marked Equation 1, Equation 2, and so on) almost always score better than students whose work is muddled. Examiners can only credit what they can read.

Notes from the examiner's report

There were a fair number of blank responses on this high-grade question. Most who attempted it earned at least the first method mark by setting up an equation from $S_{30} = 4395$. The trickier step was correctly forming the second equation involving $u_{10} + u_{20}$. Once both equations were set up, the simultaneous solving was usually straightforward.

Total: M1 + M1 + M1 + M1 + A1 = 5 marks.

Question 20 Tangent at the minimum point of a quartic (4 marks)

$y = 2x^4 - 64x$. Differentiate: **[M1 (for differentiating one term correctly)]**

$$\frac{dy}{dx} = 8x^3 - 64.$$

At a stationary point, $\frac{dy}{dx} = 0$: **[M1 dep (for setting $8x^3 - 64 = 0$ or equivalent)]**

$$8x^3 - 64 = 0 \implies x^3 = 8 \implies x = 2.$$

[M1 dep (for solving for x)]

Find the y-coordinate: $y = 2(2)^4 - 64(2) = 32 - 128 = -96.$

Find the equation of the tangent. At any minimum (or maximum) the tangent is *horizontal*, because $\frac{dy}{dx} = 0$ there. So the tangent is:

$$\boxed{y = -96}.$$

[A1 (must be an equation in y)]

Mathsaurus tip: Horizontal tangent at a stationary point

Don't be tempted into the standard $y - y_1 = m(x - x_1)$ formula and lose the simplicity. When the gradient is 0, the tangent is just $y =$ (the y-coordinate of the point).

Notes from the examiner's report

Two traps to avoid here. First, when cube-rooting $x^3 = 8$, the answer is $x = 2$ only (*not* ± 2, since cube roots are unique). Second, the question asks for the *equation* of the tangent. Writing $(2, -96)$ on the answer line names the wrong object and loses the final mark. Some students worked out $y = -96$ correctly then added spurious extra working, which lost them the mark.

Total: M1 + M1 + M1 + A1 = 4 marks.

Question 21 Upper bound of a calculation (3 marks)

$$T = \frac{x^2 + y^2}{w}, \qquad x = 28.4 \text{ (1dp)}, \quad y = 17 \text{ (2sf)}, \quad w = 90 \text{ (nearest 5)}.$$

Step 1: write down the bounds for each variable. **[B1 (for at least one correct upper or lower bound from $28.45,\ 17.5,\ 87.5$)]**

$$28.35 \le x < 28.45, \qquad 16.5 \le y < 17.5, \qquad 87.5 \le w < 92.5.$$

Step 2: decide which bounds to use for an upper bound of T.

To make T as large as possible:

- the numerator $x^2 + y^2$ should be *as large as possible* $\Rightarrow$ use upper bounds: $x = 28.45$, $y = 17.5$.
- the denominator w should be *as small as possible* $\Rightarrow$ use the lower bound $w = 87.5$.

Step 3: compute. **[M1 (for substituting all three correct bounds into the formula)]**

$$T_{\text{upper}} = \frac{(28.45)^2 + (17.5)^2}{87.5} = \frac{809.4025 + 306.25}{87.5} = \frac{1115.6525}{87.5} \approx 12.7503.$$

To 3 sf:

$$\boxed{12.8}.$$

[A1 awrt 12.8]

Mathsaurus tip: Max numerator, min denominator

Why is the lower bound of w (the smaller value 87.5) what we want for an *upper* bound of T? Because dividing by a *smaller* number gives a *larger* answer.

Notes from the examiner's report

A common method error: substituting the given values $x = 28.4$, $y = 17$, $w = 90$ directly into the formula and *then* trying to find bounds afterwards. This gains no marks. The bounds must be substituted from the start. Some students used 28.449... recurring instead of 28.45 (and similar for 17); 28.45 is the correct upper bound for a value rounded to 1 dp.

Total: B1 + M1 + A1 = 3 marks.

Question 22 Volume of metal in a hemispherical bowl (3 marks)

The bowl is hollow: solid metal between an outer hemisphere ($r = 12$ cm) and an inner hemisphere ($r = 9$ cm). The volume of metal is the difference.

Volume of a sphere is $\frac{4}{3}\pi r^3$, so a hemisphere is half of that, $\frac{2}{3}\pi r^3$: **[M1 (for finding a sphere or hemisphere volume with $r = 12$ or $r = 9$)]**

$$V_{\text{outer}} = \tfrac{2}{3}\pi(12)^3 = \tfrac{2}{3}\pi(1728) = 1152\pi.$$

$$V_{\text{inner}} = \tfrac{2}{3}\pi(9)^3 = \tfrac{2}{3}\pi(729) = 486\pi.$$

Subtract: **[M1 (for a complete method: subtract and halve)]**

$$V_{\text{metal}} = 1152\pi - 486\pi = 666\pi \approx 2092.04 \text{ cm}^3.$$

$$\boxed{2092 \text{ cm}^3}.$$

[A1 (allow 2091 – 2093)]

Mathsaurus tip: Keep π until the end

Keep the answer in terms of π (here 666π) until the very last step. Multiplying by π once at the end is more accurate and avoids rounding errors in the middle.

Notes from the examiner's report

The single most common slip: working out $\frac{4}{3}\pi(12^3 - 9^3)$ for the difference of *full* spheres, then forgetting to halve. The bowl is a hemisphere, so the final answer needs to be divided by 2.

Total: M1 + M1 + A1 = 3 marks.

Question 23 Curve and horizontal line, find k (5 marks)

The curve $C : y = x^2 - 8x - 9$ meets the horizontal line $L : y = k$ at points $A(p, k)$ and $B(q, k)$. Given that $p - q = 14$, find k.

This is a five-mark question that students attempted in lots of different ways, so we'll show three valid routes. Pick whichever feels most natural; all three are credited by the official mark scheme.

Method 1: substitute one variable into the equation

Both $A(p, k)$ and $B(q, k)$ lie on the curve, so substituting each into $y = x^2 - 8x - 9$:

$$p^2 - 8p - 9 = k \qquad \text{and} \qquad q^2 - 8q - 9 = k.$$

[M1 (for both equations, or equivalent)]

The given condition $p - q = 14$ means $p = q + 14$. Substitute that into the first equation and equate the two expressions for k:

$$(q + 14)^2 - 8(q + 14) - 9 = q^2 - 8q - 9.$$

Expand the left-hand side:

$$(q^2 + 28q + 196) - (8q + 112) - 9 = q^2 - 8q - 9.$$

$$q^2 + 20q + 75 = q^2 - 8q - 9.$$

[M1 (for an equation in one variable)]

The q^2 terms cancel:

$$20q + 75 = -8q - 9 \implies 28q = -84 \implies q = -3.$$

[A1 (for $q = -3$ or $p = 11$)]

So $p = q + 14 = 11$. Substitute either back into the curve to find k: **[M1 (for substituting p or q back)]**

$$k = (-3)^2 - 8(-3) - 9 = 9 + 24 - 9 = \boxed{k = 24}.$$

[A1 dep on M2]

Check with $p = 11$: $11^2 - 8(11) - 9 = 121 - 88 - 9 = 24$. ✓

Method 2 (alternative): subtract the equations

Starting from the same two equations as Method 1:

$$p^2 - 8p - 9 = k \qquad \text{and} \qquad q^2 - 8q - 9 = k.$$

[M1]

Subtract the second from the first to eliminate k:

$$p^2 - q^2 - 8(p - q) = 0.$$

Factorise $p^2 - q^2$ as a difference of two squares:

$$(p - q)(p + q) - 8(p - q) = 0 \implies (p - q)(p + q - 8) = 0.$$

[M1 (for an equation in one variable, e.g. deducing $p+q=8$)]

Since A and B are distinct points, $p \neq q$, so we must have $p+q=8$.

Combined with $p-q=14$:

$$p+q=8, \qquad p-q=14.$$

Adding: $2p=22 \Rightarrow p=11$. Subtracting: $2q=-6 \Rightarrow q=-3$. **[A1]**

Substitute back: $k=(-3)^2-8(-3)-9=24$. **[M1, A1]**

$$\boxed{k=24}.$$

Method 3 (alternative): completing the square

Step 1: complete the square. Rewrite the curve: **[M1]**

$$x^2-8x-9=(x-4)^2-16-9=(x-4)^2-25.$$

Step 2: solve for x in terms of k. Set equal to k:

$$(x-4)^2-25=k \quad\Longrightarrow\quad (x-4)^2=k+25 \quad\Longrightarrow\quad x=4\pm\sqrt{k+25}.$$

[M1]

So $p=4+\sqrt{k+25}$ and $q=4-\sqrt{k+25}$ (taking $p>q$).

Step 3: use $p-q=14$.

$$p-q=2\sqrt{k+25}=14.$$

[A1]

Step 4: solve for k. **[M1]**

$$\sqrt{k+25}=7 \quad\Longrightarrow\quad k+25=49 \quad\Longrightarrow\quad \boxed{k=24}.$$

[A1 dep on M2]

Mathsaurus tip: Why completing the square works

Completing the square turns $y=x^2-8x-9$ into $y=(x-4)^2-25$, which immediately reveals the parabola's geometry: its axis of symmetry is $x=4$ and its minimum value is -25. Square-rooting gives the two intersection points as $4\pm\sqrt{k+25}$, and the gap $p-q$ falls out in a single line.

Notes from the examiner's report

This question stalled a lot of students; the examiner's report described it as challenging with many blank responses. The most common partial progress was finding $x=4$ (the axis of symmetry) by differentiation, then getting stuck. Don't leave a question blank just because you can't see the whole route from the start; partial method earns method marks.

Total: M1 + M1 + A1 + M1 + A1 = 5 marks.

Question 24 Vector geometry: meeting point of two lines (6 marks)

(i) Find $\overrightarrow{AQ}$. Travel from A via O to Q:

$$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -10\mathbf{a} + \tfrac{1}{5}(10\mathbf{b}) = \boxed{2\mathbf{b} - 10\mathbf{a}}.$$

[B1 (must be simplified)]

(ii) Find $\overrightarrow{OP}$. From O, go to A first, then a quarter of the way along AB:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\mathbf{b} - 10\mathbf{a}, \quad \overrightarrow{AP} = \tfrac{1}{4}(10\mathbf{b} - 10\mathbf{a}).$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 10\mathbf{a} + \tfrac{1}{4}(10\mathbf{b} - 10\mathbf{a}) = \boxed{\tfrac{15}{2}\mathbf{a} + \tfrac{5}{2}\mathbf{b}}.$$

[B1 oe (must be simplified)]

(iii) Find $\overrightarrow{OR}$.

The point R lies on *both* lines ORP and ARQ, so we set up two expressions for $\overrightarrow{OR}$ and equate them.

From line ORP: R is somewhere on OP, so $\overrightarrow{OR} = k\,\overrightarrow{OP}$ for some scalar k: **[M1 ft from (ii)]**

$$\overrightarrow{OR} = k\left(\tfrac{15}{2}\mathbf{a} + \tfrac{5}{2}\mathbf{b}\right) = \tfrac{15k}{2}\mathbf{a} + \tfrac{5k}{2}\mathbf{b}. \quad (\star)$$

From line ARQ: R is on AQ, so $\overrightarrow{OR} = \overrightarrow{OA} + \lambda\,\overrightarrow{AQ}$ for some scalar λ: **[M1 ft from (i)]**

$$\overrightarrow{OR} = 10\mathbf{a} + \lambda(2\mathbf{b} - 10\mathbf{a}) = (10 - 10\lambda)\mathbf{a} + 2\lambda\mathbf{b}. \quad (\star\star)$$

Equate a and b components from $(\star)$ and $(\star\star)$: **[M1 (for two correct equations in k and λ)]**

$$\tfrac{15k}{2} = 10 - 10\lambda \qquad \text{and} \qquad \tfrac{5k}{2} = 2\lambda.$$

From the second equation, $\lambda = \frac{5k}{4}$. Substitute into the first:

$$\tfrac{15k}{2} = 10 - 10 \times \tfrac{5k}{4} = 10 - \tfrac{25k}{2}.$$

$$\tfrac{15k}{2} + \tfrac{25k}{2} = 10 \implies 20k = 10 \implies k = \tfrac{1}{2}.$$

Sub back into $(\star)$:

$$\overrightarrow{OR} = \tfrac{1}{2}\left(\tfrac{15}{2}\mathbf{a} + \tfrac{5}{2}\mathbf{b}\right) = \boxed{\tfrac{15}{4}\mathbf{a} + \tfrac{5}{4}\mathbf{b}}.$$

[A1 oe]

Mathsaurus tip: Two paths through one point

To find where two lines meet, write the position vector of that point in two different ways (one along each line, each with its own scalar parameter). Then equate **a**- and **b**-components separately. Two equations, two unknowns.

Notes from the examiner's report

Most students managed parts (i) and (ii). The biggest pitfalls in (iii) were sign errors, missing brackets, and inconsistent vector notation. Some students wrote vectors equal to scalars, or appeared to "divide" one vector by another. Keep your scalar parameters (k, λ) clearly separate from your vectors (**a**, **b**), and check your simplification at every line.

Total: B1 + B1 + M1 + M1 + M1 + A1 = 6 marks.

Question 25 Inverse function via completing the square (4 marks)

$f(x) = 2x^2 - 24x + 7$ for $x \geq 6$.

Step 1: write $y = f(x)$ and aim to make x the subject. **[M1 (for a correct first step: completing the square or solving as a quadratic in x)]**

$$y = 2x^2 - 24x + 7.$$

Step 2: complete the square. Pull out the 2: **[M1 dep (for fully completing the square)]**

$$y = 2(x^2 - 12x) + 7 = 2\big[(x-6)^2 - 36\big] + 7 = 2(x-6)^2 - 72 + 7 = 2(x-6)^2 - 65.$$

Step 3: rearrange to make x the subject. **[M1 (for isolating $(x-6)^2$)]**

$$y + 65 = 2(x-6)^2 \quad \Longrightarrow \quad (x-6)^2 = \frac{y+65}{2}.$$

$$x - 6 = \pm\sqrt{\frac{y+65}{2}}.$$

Step 4: pick the correct sign using the domain. We are told $x \geq 6$, so $x - 6 \geq 0$. Take the positive root only:

$$x = 6 + \sqrt{\frac{y+65}{2}}.$$

Finally, swap x and y to write the inverse as a function of x:

$$\boxed{f^{-1}(x) = 6 + \sqrt{\tfrac{x+65}{2}}}.$$

[A1 (must be in x; $\pm$ form scores M3A0)]

Mathsaurus tip: Why the domain matters

The condition $x \geq 6$ in the original function isn't decoration. It makes f one-to-one (a quadratic is otherwise two-to-one), and it tells you which branch of the square root to keep when inverting.

Notes from the examiner's report

The examiner described this as "poorly attempted"; only the most able students secured full marks. Two main pitfalls. First, many didn't realise that completing the square was the route to making x the subject. Second, of those who did, many gave their final answer as $6 \pm \sqrt{\frac{x+65}{2}}$ without recognising that the domain restriction $x \geq 6$ tells you to keep only the positive root. The $\pm$ form scored zero on the final accuracy mark.

Total: M1 + M1 + M1 + A1 = 4 marks.

End of paper. Total: 100 marks.