MATHSAURUS

AS Mathematics Exam Questions by Topic
Chapter 13c: Linearising Exponentials and Polynomials with Logarithms

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI) and Edexcel. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

OCR B MEI AS 2018 Paper 1 Question 11:

11 The intensity of the sun's radiation, y watts per square metre, and the average distance from the sun, x astronomical units, are shown in Fig. 11 for the planets Mercury and Jupiter.

	x	y
Mercury	0.3075	14 400
Jupiter	4.950	55.8

Fig. 11

The intensity y is proportional to a power of the distance x.

(i) Write down an equation for y in terms of x and two constants. **[1]**

(ii) Show that the equation can be written in the form $\ln y = a + b \ln x$. **[2]**

(iii) In the Printed Answer Booklet, complete the table for $\ln x$ and $\ln y$ correct to 4 significant figures. **[2]**

(iv) Use the values from part **(iii)** to find a and b. **[3]**

(v) Hence rewrite your equation from part **(i)** for y in terms of x, using suitable numerical values for the constants. **[2]**

(vi) Sketch a graph of the equation found in part **(v)**. **[2]**

(vii) Earth is 1 astronomical unit from the sun. Find the intensity of the sun's radiation for Earth. **[1]**

Edexcel AS 2018 Paper 1 Question 13:

13.

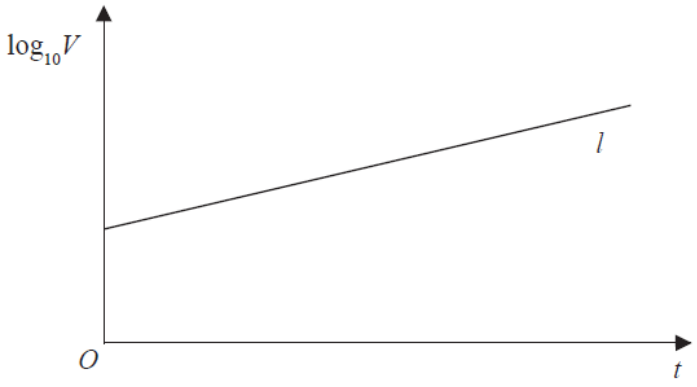


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

Edexcel AS Sample Paper 1 Question 14:

14.

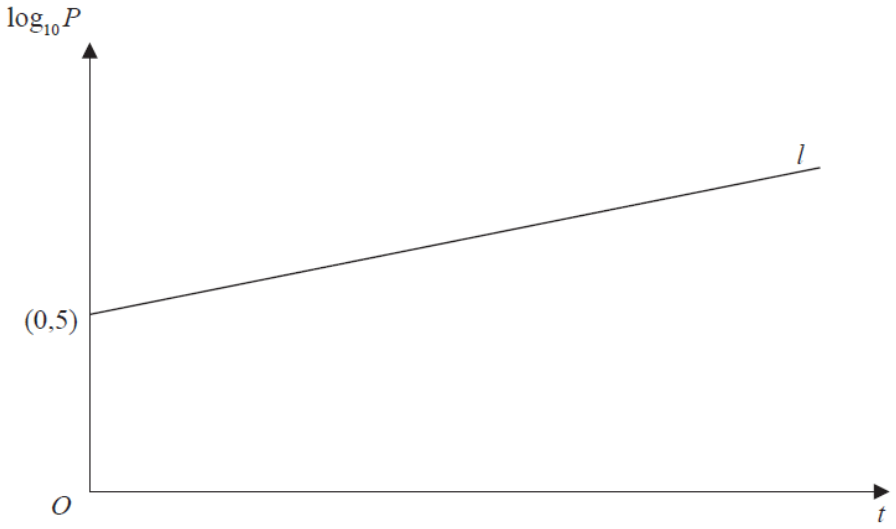


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Edexcel Sample Paper 1 Question 12:

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

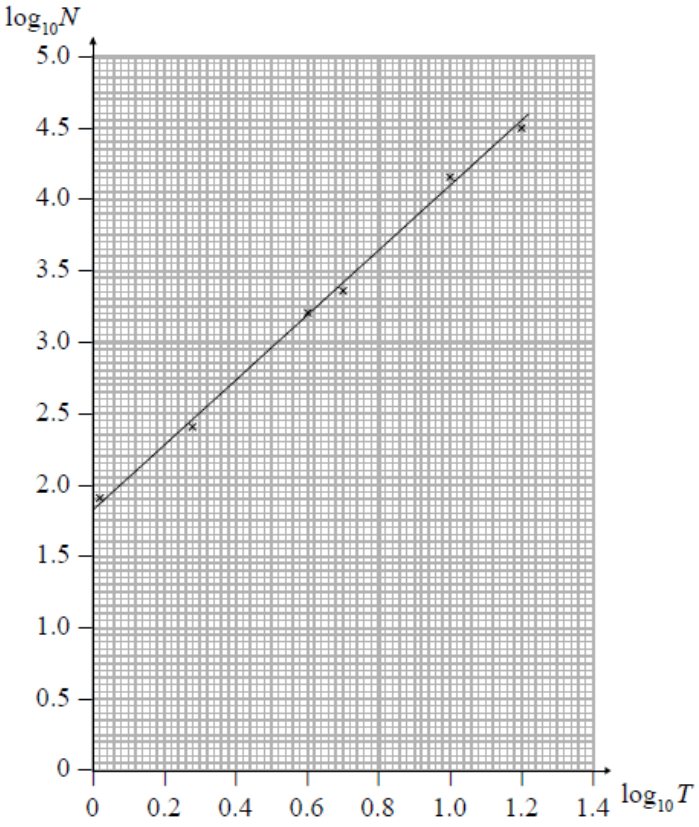


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

OCR A AS Sample Paper 2 Question 5:

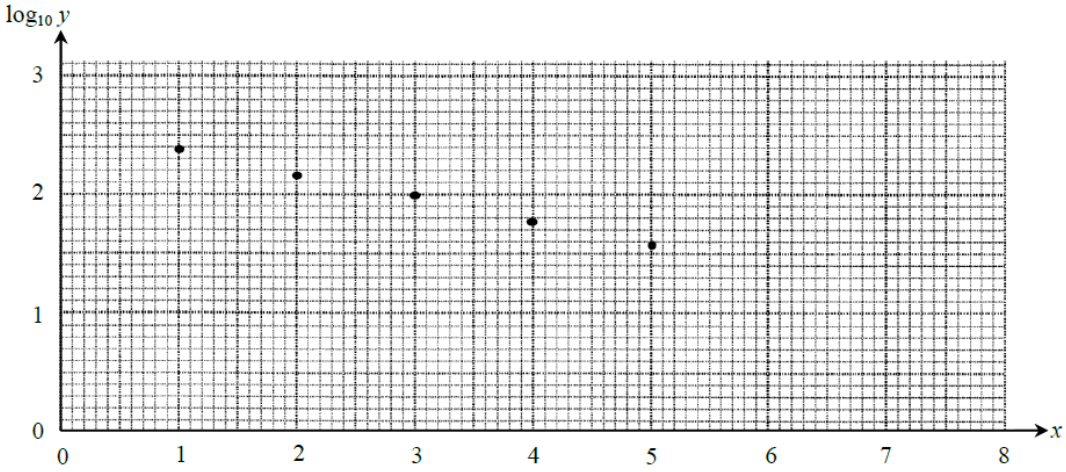
5 A doctors' surgery starts a campaign to reduce missed appointments. The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

Number of weeks after the start (x)	1	2	3	4	5
Number of missed appointments (y)	235	149	99	59	38

It was felt that this data could be modelled by an equation of the form $y = pq^x$ where p and q are constants.

(i) Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing m and c in terms of p and/or q. **[2]**

The diagram below shows $\log_{10} y$ plotted against x, for the given data.



(ii) Estimate the values of p and q. **[3]**

(iii) Use the model to predict when the number of missed appointments will fall below 20. Explain why this answer may not be reliable. **[2]**

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n, and the number of months after launch, t, can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. **[2]**

(ii) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

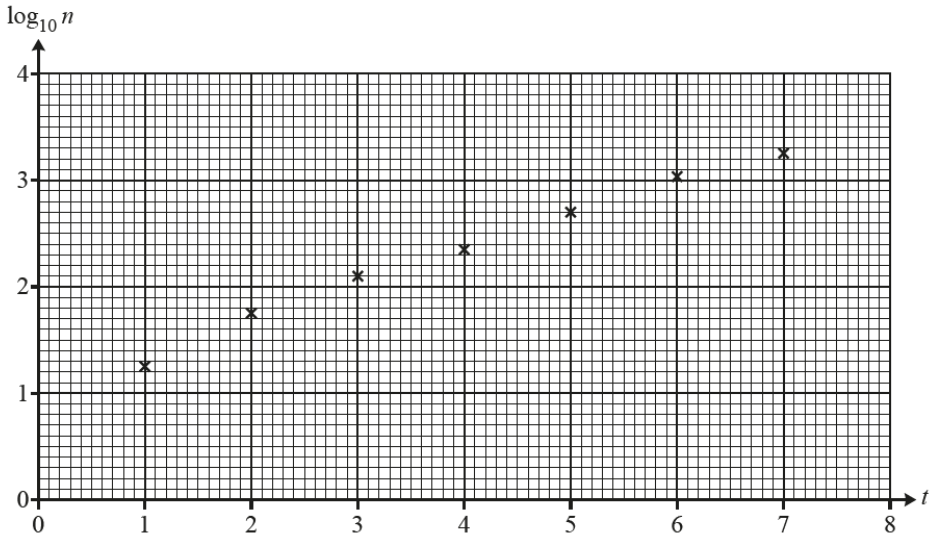


Fig. 5

Find estimates of the values of a and k. **[4]**

(iii) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. **[4]**

(iv) Give a reason why the model may not be appropriate for large values of t. **[1]**