MATHSAURUS

A Level Mathematics Year 2 Exam Questions by Topic
Chapter 9: Further differentiation

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI) and Edexcel. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

Edexcel Sample Paper 2 Question 5:

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

OCR A 2018 Paper 1 Question 11:

11 In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

(i) Find the time taken for the mass to decrease to half of its original value. **[3]**

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

(ii) Find the time at which both substances are decaying at the same rate. **[8]**

OCR A 2018 Paper 2 Question 6:

6 **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

(i) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

(ii) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

(iii) Find the coordinates of the turning point on the curve. **[4]**

(iv) Determine whether this turning point is a maximum or a minimum. **[5]**

OCR A Sample Paper 1 Question 10:

10 A curve has equation $x = (y+5)\ln(2y-7)$.

(i) Find $\frac{dx}{dy}$ in terms of y. **[3]**

(ii) Find the gradient of the curve where it crosses the y-axis. **[5]**

OCR A Sample Paper 1 Question 13:

13 **In this question you must show detailed reasoning.**

Find the exact values of the x-coordinates of the stationary points of the curve $x^3 + y^3 = 3xy + 35$. **[9]**

OCR B MEI 2018 Paper 2 Question 15:

15 **You must show detailed reasoning in this question.**

The equation of a curve is

$$y^3 - xy + 4\sqrt{x} = 4 .$$

Find the gradient of the curve at each of the points where $y = 1$. **[9]**

OCR B MEI Sample Paper 2 Question 12:

12 Fig. 12 shows the curve $2x^3 + y^3 = 5y$.

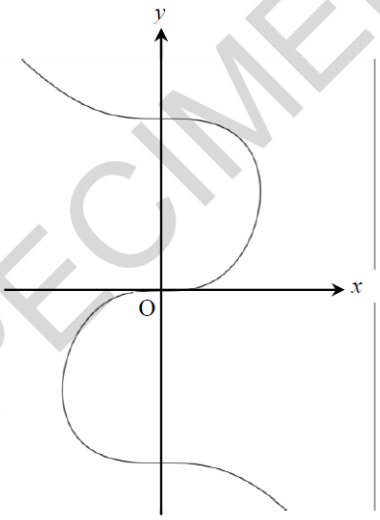

Fig. 12

(i) Find the gradient of the curve $2x^3 + y^3 = 5y$ at the point $(1, 2)$, giving your answer in exact form. **[4]**

(ii) Show that all the stationary points of the curve lie on the y-axis. **[2]**

Edexcel 2018 Paper 1 Question 5:

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

Edexcel 2018 Paper 1 Question 9:

9.

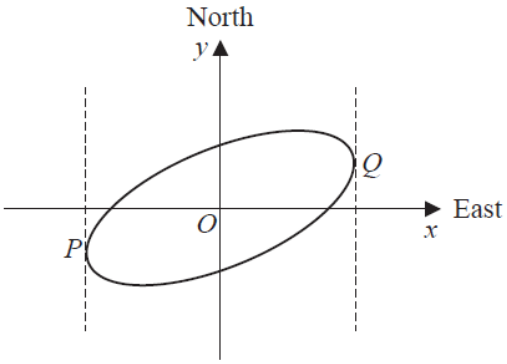

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

Edexcel 2018 Paper 2 Question 9:

9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Edexcel 2018 Paper 2 Question 14:

14. A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

Edexcel Sample Paper 1 Question 10:

10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Edexcel Sample Paper 1 Question 15:

15.

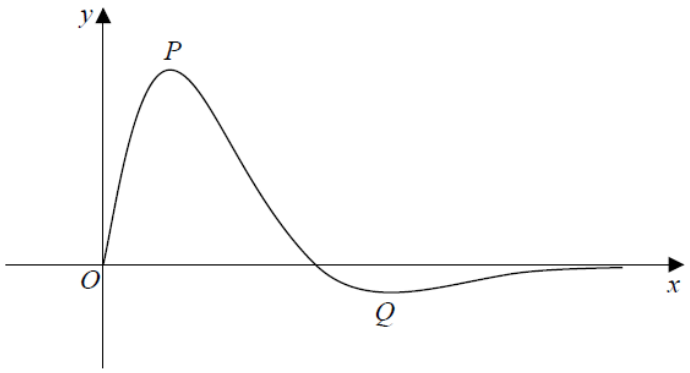

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)