

AS Mathematics Exam Questions by Topic Chapter 13b: Exponential Modelling

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: <https://amzn.to/39umfr5> and <https://amzn.to/3hE8kBL>). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

<https://mathsaurus.com/>

OCR B MEI AS Sample Paper 1 Question 9:

- 9 A biologist is investigating the growth of bacteria in a piece of bread. He believes that the number, N , of bacteria after t hours may be modelled by the relationship $N = A \times 2^{kt}$, where A and k are constants.
- (i) Show that, according to the model, the graph of $\log_{10} N$ against t is a straight line.
Give, in terms of A and k ,
- the gradient of the line
 - the intercept on the vertical axis. [4]

The biologist measures the number of bacteria at regular intervals over 22 hours and plots a graph of $\log_{10} N$ against t . He finds that the graph is approximately a straight line with gradient 0.20; the line crosses the vertical axis at 2.0.

- (ii) Find the values of A and k . [2]
- (iii) Use the model to predict the number of bacteria after 24 hours. [1]
- (iv) Give a reason why the model may not be appropriate for large values of t . [1]
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OCR B MEI AS Sample Paper 2 Question 8:

- 8 In an experiment, the temperature of a hot liquid is measured every minute. The difference between the temperature of the hot liquid and room temperature is D °C at time t minutes.

Fig. 8 shows the experimental data.

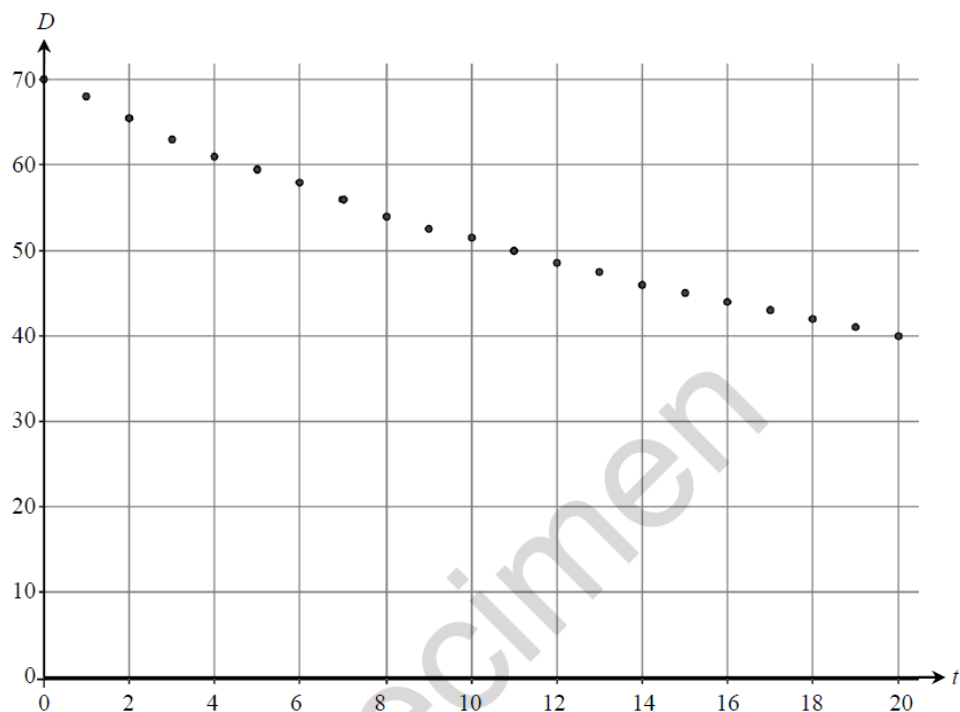


Fig. 8

It is thought that the model $D = 70e^{-0.03t}$ might fit the data.

- (i) Write down the derivative of $e^{-0.03t}$. [1]
- (ii) Explain how you know that $70e^{-0.03t}$ is a decreasing function of t . [1]
- (iii) Calculate the value of $70e^{-0.03t}$ when
- (A) $t = 0$, [1]
- (B) $t = 20$. [1]
- (iv) Using your answers to parts (ii) and (iii), discuss how well the model $D = 70e^{-0.03t}$ fits the data. [3]
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AQA Sample Paper 2 Question 2:

- 2 A zoologist is investigating the growth of a population of red squirrels in a forest.

She uses the equation $N = \frac{200}{1 + 9e^{-\frac{t}{5}}}$ as a model to predict the number of squirrels,

N , in the population t weeks after the start of the investigation.

What is the size of the squirrel population at the start of the investigation?

Circle your answer.

[1 mark]

5

20

40

200

OCR B MEI Sample Paper 1 Question 10:

- 10 In a certain region, the populations, P_G and P_R , of grey and red squirrels at time t years are modelled by the equations

$$P_G = 10000(1 - e^{-kt})$$

$$P_R = 20000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

- (i) (A) On the axes in your Printed Answer Book, sketch the graphs of P_G and P_R on the same axes.

(B) Give the equations of any asymptotes. [4]

- (ii) What does the model predict about the long term population of

- grey squirrels
- red squirrels?

[2]

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful.

- (iii) Comment on the validity of the model given by the equations, giving a reason for your answer. [1]

- (iv) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. [4]

- (v) Given that the numbers of grey and red squirrels are equal when $t = 3$, find the value of k . [4]
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AQA 2018 Paper 1 Question 10:

- 10** A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using

$$m = m_0 e^{-kt}$$

where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

On average, it takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

- 10 (a)** The scientist drinks two strong cups of coffee at 8 am. Use the model to estimate the mass of caffeine in the scientist's body at midday.

[4 marks]

- 10 (b)** The scientist wants the mass of caffeine in her body to stay below 480 mg

Use the model to find the earliest time that she could drink another cup of strong coffee.

Give your answer to the nearest minute.

[3 marks]

- 10 (c)** State a reason why the mass of caffeine remaining in the scientist's body predicted by the model may not be accurate.

[1 mark]

AQA AS 2018 Paper 2 Question 12:

- 12** Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_A , can be modelled by the formula

$$n_A = ae^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_B , can be modelled by the formula

$$n_B = be^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

- 12 (a)** Show that $b = 90$, to the nearest integer, and find the value of a . **[3 marks]**

- 12 (b)** Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020. **[1 mark]**

- 12 (c)** Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A. **[3 marks]**

- 12 (d)** Comment on the long-term accuracy of the model. **[1 mark]**
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AQA AS Sample Paper 2 Question 10:

- 10** David has been investigating the population of rabbits on an island during a three-year period.

Based on data that he has collected, David decides to model the population of rabbits, R , by the formula

$$R = 50e^{0.5t}$$

where t is the time in years after 1 January 2016.

- 10 (a)** Using David's model:

- 10 (a) (i)** state the population of rabbits on the island on 1 January 2016;

[1 mark]

- 10 (a) (ii)** predict the population of rabbits on 1 January 2021.

[1 mark]

- 10 (b)** Use David's model to find the value of t when $R = 150$, giving your answer to three significant figures.

[2 marks]

- 10 (c)** Give **one** reason why David's model may **not** be appropriate.

[1 mark]

- 10 (d)** On the same island, the population of crickets, C , can be modelled by the formula

$$C = 1000e^{0.1t}$$

where t is the time in years after 1 January 2016.

Using the two models, find the year during which the population of rabbits first exceeds the population of crickets.

[3 marks]

Edexcel 2018 Paper 1 Question 12:

12. The value, $\pounds V$, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was $\pounds 32\,000$ on 1st January 2005 and $\pounds 50\,000$ on 1st January 2012

- (a) (i) find p to 4 decimal places,
(ii) show that A is approximately 24 800 (4)
- (b) With reference to the model, interpret
(i) the value of the constant A ,
(ii) the value of the constant p . (2)

Using the model,

- (c) find the year during which the value of the car first exceeds $\pounds 100\,000$ (4)
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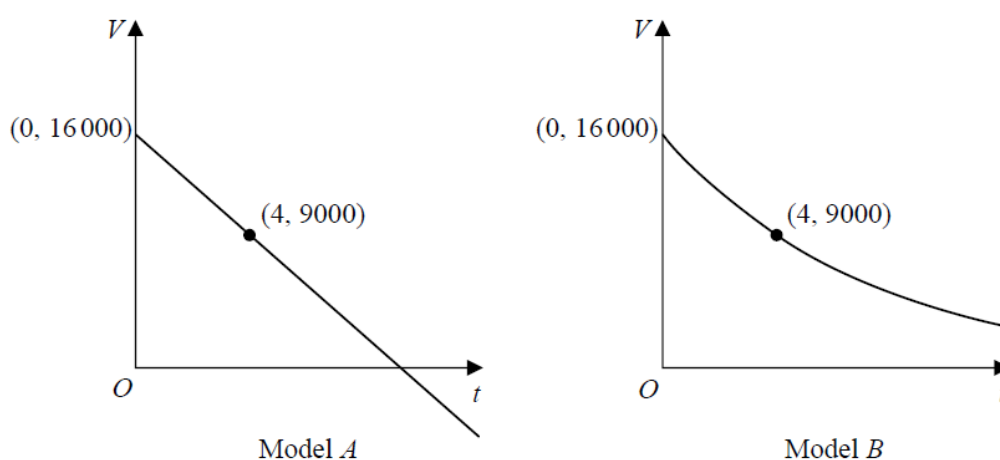
6. A company plans to extract oil from an oil field.

The daily volume of oil V , measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



- (a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.
- (ii) Write down a limitation of using model A . (2)
- (b) (i) Using an exponential model and the information given in the question, find a possible equation for model B .
- (ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling. (5)
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OCR A AS 2018 Paper 2 Question 6:

- 6 A pan of water is heated until it reaches 100°C . Once the water reaches 100°C , the heat is switched off and the temperature $T^{\circ}\text{C}$ of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

- (i) Write down the value of a . [1]

- (ii) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15°C per minute.

- (iii) Calculate the value of k . [3]

- (iv) Find the time taken for the temperature of the water to drop from 100°C to 45°C . [3]

- (v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100°C . Suggest how the equation for the temperature as the water cools would be modified by this. [1]

OCR A AS Sample Paper 2 Question 3:

- 3 The number of members of a social networking site is modelled by $m = 150e^{2t}$, where m is the number of members and t is time in weeks after the launch of the site.

- (i) State what this model implies about the relationship between m and the rate of change of m . [2]

- (ii) What is the significance of the integer 150 in the model? [1]

- (iii) Find the week in which the model predicts that the number of members first exceeds 60 000. [3]

- (iv) The social networking site only expects to attract 60 000 members. Suggest how the model could be refined to take account of this. [1]

OCR B MEI 2018 Paper 2 Question 16:

- 16** In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for t hours. Over the course of the year she realises that her percentage mark for a test, p , may be modelled by the following formula, where A , B and C are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.
- Aaishah had a mark of 22 when she didn't spend any time revising.

(i) Find the values of A , B and C . [3]

(ii) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve? [2]

(iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work? [2]

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p .

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

(iv) Verify that the data is consistent with the new formula. [2]

(v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. [2]

OCR B MEI AS 2018 Paper 2 Question 12:

- 12 In an experiment 500 fruit flies were released into a controlled environment. After 10 days there were 650 fruit flies present.

Munirah believes that N , the number of fruit flies present at time t days after the fruit flies are released, will increase at the rate of 4.4% per day. She proposes that the situation is modelled by the formula $N = Ak^t$.

- (i) Write down the values of A and k . [2]
- (ii) Determine whether the model is consistent with the value of N at $t = 10$. [2]
- (iii) What does the model suggest about the number of fruit flies in the long run? [1]

Subsequently it is found that for large values of t the number of fruit flies in the controlled environment oscillates about 750. It is also found that as t increases the oscillations decrease in magnitude.

Munirah proposes a second model in the light of this new information.

$$N = 750 - 250 \times e^{-0.092t}$$

- (iv) Identify three ways in which this second model is consistent with the known data. [3]
- (v) (A) Identify one feature which is not accounted for by the second model. [1]
- (B) Give an example of a mathematical function which needs to be incorporated in the model to account for this feature. [1]

AQA AS Sample Paper 2 Question 9:

- 9 A curve has equation $y = e^{2x}$

Find the coordinates of the point on the curve where the gradient of the curve is $\frac{1}{2}$

Give your answer in an exact form.

[5 marks]

15.

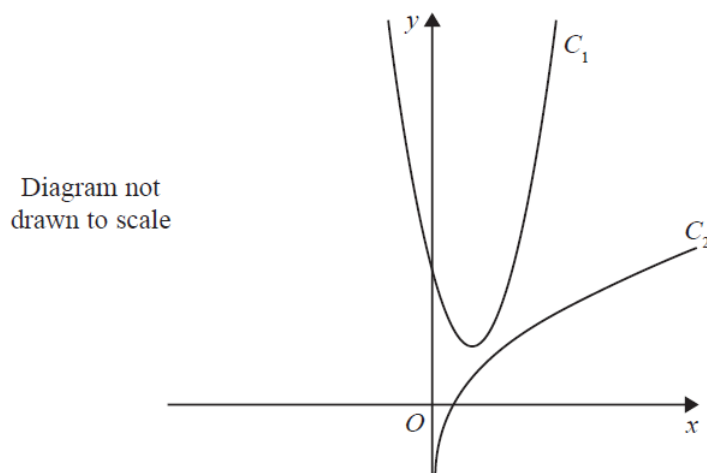


Figure 3

The curve C_1 , shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2 , also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q .

Find the exact coordinates of Q .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(8)