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**AS Mathematics Exam Questions by Topic**  
**Chapter 1: Problem Solving**

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These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: <https://amzn.to/39umfr5> and <https://amzn.to/3hE8kBL> ). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

<https://mathsaurus.com/>

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AQA 2018 Paper 2 Question 1:

**1** Which of these statements is correct?

Tick **one** box.

[1 mark]

$$x = 2 \Rightarrow x^2 = 4$$

$$x^2 = 4 \Rightarrow x = 2$$

$$x^2 = 4 \Leftrightarrow x = 2$$

$$x^2 = 4 \Rightarrow x = -2$$

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AQA 2018 Paper 2 Question 5:

**5** Prove that 23 is a prime number.

[2 marks]

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AQA AS 2018 Paper 1 Question 7:

7 Prove that

$n$  is a prime number greater than 5  $\Rightarrow n^4$  has final digit 1

[5 marks]

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AQA AS Sample Paper 1 Question 2:

2 Consider the two statements, A and B, below.

A:  $x^2 - 6x + 8 > 0$

B:  $x > 4$

Choose the most appropriate option below.

Circle your answer.

[1 mark]

$A \Rightarrow B$

$A \Leftarrow B$

$A \Leftrightarrow B$

There is no  
connection  
between A and  
B

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AQA AS Sample Paper 2 Question 12:

12 (a) Given that  $n$  is an even number, prove that  $9n^2 + 6n$  has a factor of 12

[3 marks]

12 (b) Determine if  $9n^2 + 6n$  has a factor of 12 for any integer  $n$ .

[1 mark]

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OCR A 2018 Paper 1 Question 4:

4 Prove algebraically that  $n^3 + 3n - 1$  is odd for all positive integers  $n$ .

[4]

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AQA Sample Paper 2 Question 7:

- 7 A student notices that when he adds two consecutive odd numbers together the answer always seems to be the difference between two square numbers.

He claims that this will always be true.

He attempts to prove his claim as follows:

**Step 1:** Check first few cases

$$3 + 5 = 8 \text{ and } 8 = 3^2 - 1^2$$

$$5 + 7 = 12 \text{ and } 12 = 4^2 - 2^2$$

$$7 + 9 = 16 \text{ and } 16 = 5^2 - 3^2$$

**Step 2:** Use pattern to predict and check a large example

$$101 + 103 = 204$$

subtract 1 and divide by 2 for the first number

Add 1 and divide by two for the second number

$$52^2 - 50^2 = 204 \text{ it works!}$$

**Step 3:** Conclusion

The first few cases work and there is a pattern, which can be used to predict larger numbers.

Therefore, it must be true for all consecutive odd numbers.

- 7 (a) Explain what is wrong with the student's "proof". [1 mark]

- 7 (b) Prove that the student's claim is correct. [3 marks]

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Edexcel AS Sample Paper 1 Question 11:

11. (a) Prove that for all positive values of  $x$  and  $y$

$$\sqrt{xy} \leq \frac{x + y}{2} \quad (2)$$

- (b) Prove by counter example that this is not true when  $x$  and  $y$  are both negative. (1)

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OCR A AS 2018 Paper 1 Question 5:

- 5  $N$  is an integer that is not divisible by 3. Prove that  $N^2$  is of the form  $3p + 1$ , where  $p$  is an integer. [5]

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OCR A AS 2018 Paper 2 Question 3:

- 3 In each of the following cases choose one of the statements

$$P \Rightarrow Q \quad P \Leftarrow Q \quad P \Leftrightarrow Q$$

to describe the relationship between  $P$  and  $Q$ .

- (i)  $P: y = 3x^5 - 4x^2 + 12x$   
 $Q: \frac{dy}{dx} = 15x^4 - 8x + 12$  [1]
- (ii)  $P: x^5 - 32 = 0$  where  $x$  is real  
 $Q: x = 2$  [1]
- (iii)  $P: \ln y < 0$   
 $Q: y < 1$  [1]

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OCR A AS Sample Paper 2 Question 6:

- 6 (i) A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is odd. For example, 23 has digits 2 and 3 which gives  $2^2 + 3^2 = 13$ , which is odd. Show by counter example that this suggestion is false. [2]
- (ii) Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3. [3]

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OCR B MEI AS 2018 Paper 2 Question 3:

- 3  $P$  and  $Q$  are consecutive **odd** positive integers such that  $P > Q$ .  
Prove that  $P^2 - Q^2$  is a multiple of 8. [3]

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OCR B MEI Sample Paper 3 Question 7:

- 7 By finding a counter example, disprove the following statement.

If  $p$  and  $q$  are non-zero real numbers with  $p < q$ , then  $\frac{1}{p} > \frac{1}{q}$ . [2]

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Edexcel Sample Paper 2 Question 6:

6. Complete the table below. The first one has been done for you.

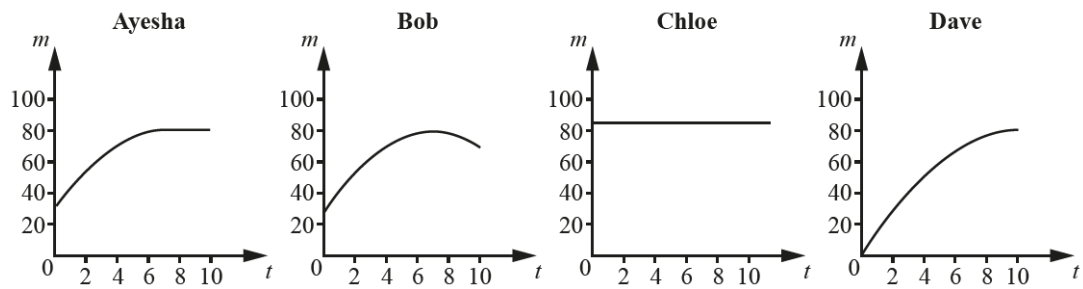
For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$ . When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive.  (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$  (2)				
(iii) The difference between consecutive square numbers is odd.  (2)				

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OCR A 2018 Paper 2 Question 3:

- 3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time,  $t$  hours, they might spend revising for an examination, and the mark,  $m$ , they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



- (i) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? [1]
- (ii) State one feature of Dave's model that is likely to be unrealistic. [1]
- (iii) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. [1]
- (iv) What does Chloe's model suggest about her attitude to revision? [1]
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