MATHSAURUS

AS Mathematics Exam Questions by Topic
Chapter 21: Variable acceleration

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

AQA AS Sample Paper 1 Question 16:

16 A particle, of mass 400 grams, is initially at rest at the point O.

The particle starts to move in a straight line so that its velocity, v m s^{-1}, at time t seconds is given by

$$v = 6t^2 - 12t^3 \text{ for } t > 0$$

16 (a) Find an expression, in terms of t, for the force acting on the particle.

[3 marks]

16 (b) Find the time when the particle next passes through O.

[5 marks]

OCR B MEI AS Sample Paper 1 Question 5:

5 A particle P moves on a straight line that contains the point O. At time t seconds the displacement of P from O is s metres, where $s = t^3 - 3t^2 + 3$.

(i) Determine the times when the particle has zero **velocity**. **[3]**

(ii) Find the distances of P from O at the times when it has zero velocity. **[2]**

AQA 2018 Paper 2 Question 15:

15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in $m\,s^{-2}$ of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

$$\text{The acceleration of A} = 0.138t^2$$
$$\text{The acceleration of B} = 0.024t^3$$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

Edexcel AS 2018 Paper 2 Question 8:

8. A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

AQA AS 2018 Paper 1 Question 16:

16 A remote-controlled toy car is moving over a horizontal surface. It moves in a straight line through a point A.

The toy is initially at the point with displacement 3 metres from A. Its velocity, $v\ \text{m s}^{-1}$, at time t seconds is defined by

$$v = 0.06(2 + t - t^2)$$

16 (a) Find an expression for the displacement, r metres, of the toy from A at time t seconds.

[4 marks]

16 (b) **In this question use $g = 9.8\,\text{m s}^{-2}$**

At time $t = 2$ seconds, the toy launches a ball which travels directly upwards with initial speed $3.43\,\text{m s}^{-1}$

Find the time taken for the ball to reach its highest point.

[3 marks]

Edexcel AS Sample Paper 2 Question 8:

8. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leqslant t \leqslant 10$

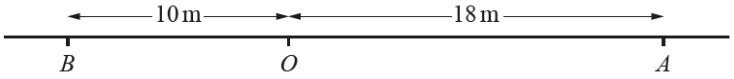
(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

OCR A AS 2018 Paper 2 Question 11:

11



A particle P is moving along a straight line with constant acceleration. Initially the particle is at O. After 9 s, P is at a point A, where $OA = 18$ m (see diagram) and the velocity of P at A is $8\,\text{m s}^{-1}$ in the direction $\overrightarrow{OA}$.

(i) **(a)** Show that the initial speed of P is $4\,\text{m s}^{-1}$. **[2]**

(b) Find the acceleration of P. **[2]**

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

(ii) Show that P is never at point B. **[4]**

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O, and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a, b and c are constants.

It is given that
- the velocity and acceleration of Q at the point O are the same as those of P at O,
- Q reaches the point A when $t = 6$.

(iii) Find the velocity of Q at A. **[5]**

OCR A AS Sample Paper 2 Question 10:

10 A student is attempting to model the flight of a boomerang. She throws the boomerang from a fixed point O and catches it when it returns to O. She suggests the model for the displacement, s metres, after t seconds is given by $s = 9t^2 - \frac{3}{2}t^3$, $0 \le t \le 6$. For this model

(i) determine what happens at $t = 6$, **[2]**

(ii) find the greatest displacement of the boomerang from O, **[4]**

(iii) find the velocity of the boomerang 1 second before the student catches it, **[2]**

(iv) find the acceleration of the boomerang 1 second before the student catches it. **[2]**

OCR B MEI 2018 Paper 1 Question 14:

14 The velocity of a car, $v\,\text{m s}^{-1}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\text{m s}^{-1}$ and it accelerates to $11.4\,\text{m s}^{-1}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

(ii) Explain why this model is not appropriate in the long term. **[1]**

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m s}^{-1}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

OCR A 2018 Paper 3 Question 11:

11 The velocity $v\,\text{m s}^{-1}$ of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is $1.3\,\text{m s}^{-2}$. For $t > 20$ the car continues its journey with constant acceleration $1.3\,\text{m s}^{-2}$ until its speed reaches $25\,\text{m s}^{-1}$.

(i) Find the value of k. **[3]**

(ii) Find the total distance the car has travelled when its speed reaches $25\,\text{m s}^{-1}$. **[7]**
