

AS Mathematics Exam Questions by Topic
Chapter 18: Statistical hypothesis testing using the binomial distribution

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: <https://amzn.to/39umfr5> and <https://amzn.to/3hE8kBL>). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

<https://mathsaurus.com/>

OCR B MEI AS Sample Paper 2 Question 10:

10 A company operates trains. The company claims that 92% of its trains arrive on time. You should assume that in a random sample of trains, they arrive on time independently of each other.

(i) Assuming that 92% of the company's trains arrive on time, find the probability that in a random sample of 30 trains operated by this company

(A) exactly 28 trains arrive on time, [2]

(B) more than 27 trains arrive on time. [2]

A journalist believes that the percentage of trains operated by this company which arrive on time is lower than 92%.

(ii) To investigate the journalist's belief a hypothesis test will be carried out at the 1% significance level. A random sample of 18 trains is selected. For this hypothesis test,

- state the hypotheses,
 - find the critical region. [5]
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AQA 2018 Paper 3 Question 17:

17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

AQA AS 2018 Paper 2 Question 19:

19 Martin grows cucumbers from seed.

In the past, he has found that 70% of all seeds successfully germinate and grow into cucumber plants.

He decides to try out a new brand of seed.

The producer of this brand claims that these seeds are more likely to successfully germinate than other brands of seeds.

Martin sows 20 of this new brand of seed and 18 successfully germinate.

Carry out a hypothesis test at the 5% level of significance to investigate the producer's claim.

[7 marks]

OCR A 2018 Paper 2 Question 9:

9 Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. **[7]**

AQA AS Sample Paper 2 Question 19:

19 Ellie, a statistics student, read a newspaper article that stated that 20 per cent of students eat at least five portions of fruit and vegetables every day.

Ellie suggests that the number of people who eat at least five portions of fruit and vegetables every day, in a sample of size n , can be modelled by the binomial distribution $B(n, 0.20)$.

19 (a) There are 10 students in Ellie's statistics class.

Using the distributional model suggested by Ellie, find the probability that, of the students in her class:

19 (a) (i) two or fewer eat at least five portions of fruit and vegetables every day;

[1 mark]

19 (a) (ii) at least one but fewer than four eat at least five portions of fruit and vegetables every day;

[2 marks]

19 (b) Ellie's teacher, Declan, believes that more than 20 per cent of students eat at least five portions of fruit and vegetables every day. Declan asks the 25 students in his other statistics classes and 8 of them say that they eat at least five portions of fruit and vegetables every day.

19 (b) (i) Name the sampling method used by Declan.

[1 mark]

19 (b) (ii) Describe **one** weakness of this sampling method.

[1 mark]

19 (b) (iii) Assuming that these 25 students may be considered to be a random sample, carry out a hypothesis test at the 5% significance level to investigate whether Declan's belief is supported by this evidence.

[6 marks]

AQA Sample Paper 3 Question 12:

12 During the 2006 Christmas holiday, John, a maths teacher, realised that he had fallen ill during 65% of the Christmas holidays since he had started teaching.

In January 2007, he increased his weekly exercise to try to improve his health.

For the next 7 years, he only fell ill during 2 Christmas holidays.

12 (a) Using a binomial distribution, investigate, at the 5% level of significance, whether there is evidence that John's rate of illness during the Christmas holidays had decreased since increasing his weekly exercise.

[6 marks]

12 (b) State **two** assumptions, regarding illness during the Christmas holidays, that are necessary for the distribution you have used in part (a) to be valid.

For **each** assumption, comment, in context, on whether it is likely to be correct.

[4 marks]

Edexcel AS 2018 Paper 2 Question 3:

3. Naasir is playing a game with two friends. The game is designed to be a game of chance

so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim,

the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

Edexcel AS Sample Paper 2 Question 5:

5. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geq 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

- (b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

- (c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

- (d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

- (e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

- (f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

OCR A AS 2018 Paper 1 Question 12:

- 12 It is known that 20% of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the 2% significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease. [7]

OCR A AS Sample Paper 1 Question 12:

12 It is known that under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year. In a trial of a new treatment, 450 patients with this disease were selected and the number, X , that experienced side effects within one year was noted.

It was found that 51 of the 450 patients experienced side effects within one year.

(i) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment. [7]

(ii) It was later discovered that all 450 patients selected for the trial were treated in the same hospital. Comment on the validity of the model used in part (i). [1]

OCR B MEI AS 2018 Paper 2 Question 9:

9 In this question you must show detailed reasoning.

Research showed that in May 2017 62% of adults over 65 years of age in the UK used a certain online social media platform. Later in 2017 it was believed that this proportion had increased. In December 2017 a random sample of 59 adults over 65 years of age in the UK was collected. It was found that 46 of the 59 adults used this online social media platform.

Use a suitable hypothesis test to determine whether there is evidence at the 1% level to suggest that the proportion of adults over 65 in the UK who used this online social media platform had increased from May 2017 to December 2017. [7]
