
A Level Mathematics Year 2 Exam Questions by Topic
Chapter 8: Trigonometric identities

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

Edexcel Sample Paper 1 Question 9:

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

Edexcel Sample Paper 2 Question 12:

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

Edexcel Sample Paper 2 Question 13:

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

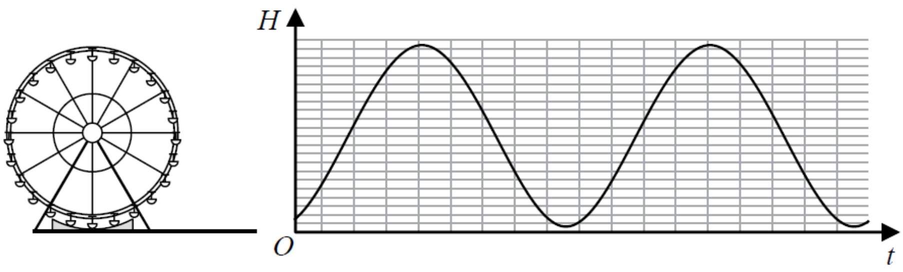


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

OCR A 2018 Paper 2 Question 4:

4 Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. **[4]**

OCR A 2018 Paper 3 Question 6:

6 It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3\cos\left(2\theta + \frac{1}{4}\pi\right)$.

(i) Show that $\tan 2\theta = \frac{1}{2}$. **[3]**

(ii) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. **[5]**

OCR A Sample Paper 1 Question 8:

8 **(i)** Show that $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$. **[3]**

(ii) In this question you must show detailed reasoning.

Solve $\dfrac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \le \theta \le \pi$. **[3]**

OCR A Sample Paper 3 Question 3:

3 **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0° < x < 90°$, find the exact value of $\sin x$. **[4]**

OCR B MEI 2018 Paper 2 Question 6:

6 **(i)** Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$. **[3]**

(ii) Write down the range of the function

$$f(x) = 12 + 7\cos x - 24\sin x, \qquad 0 \leqslant x \leqslant 2\pi.$$

[2]

OCR B MEI Sample Paper 3 Question 8:

8 In Fig. 8, OAB is a thin bent rod, with OA = 1 m, AB = 2 m and angle OAB = 120°. Angles θ, ϕ and h are as shown in Fig. 8.

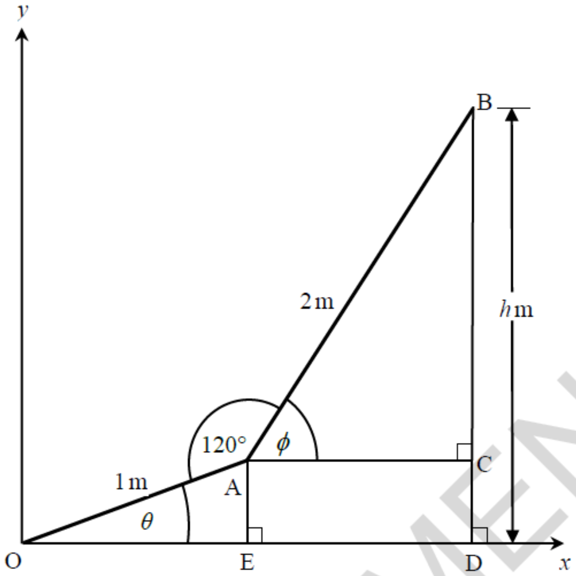


Fig. 8

(i) Show that $h = \sin\theta + 2\sin(\theta + 60°)$. **[3]**

The rod is free to rotate about the origin so that θ and ϕ vary. You may assume that the result for h in part **(i)** holds for all values of θ.

(ii) Find an angle θ for which $h = 0$. **[5]**

OCR B MEI Sample Paper 3 Question 9:

9 **(i)** Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. **[4]**

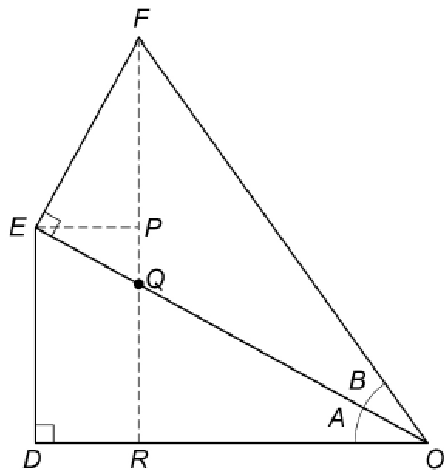
The function $f(\theta)$ is defined by $f(\theta) = \dfrac{1}{(k + \cos\theta + 2\sin\theta)}$, $0 \le \theta \le 2\pi$, k is a constant.

(ii) The maximum value of $f(\theta)$ is $\dfrac{(3+\sqrt{5})}{4}$. Find the value of k. **[3]**

AQA 2018 Paper 1 Question 14:

14 Some students are trying to prove an identity for $\sin(A+B)$.

They start by drawing two right-angled triangles *ODE* and *OEF*, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle *OFR*,

Line 1 $\sin(A+B) = \dfrac{RF}{OF}$

Line 2 $= \dfrac{RP+PF}{OF}$

Line 3 $= \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $= \dfrac{DE}{\dots\dots} \times \dfrac{\dots\dots}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= \dots\dots\dots\dots\dots + \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $= \dfrac{DE}{\dots\dots} \times \dfrac{\dots\dots}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots + \cos A \sin B$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when *A* and *B* are acute angles.

[1 mark]

14 (d) Another student claims that by replacing *B* with $-B$ in the identity for $\sin(A+B)$ it is possible to find an identity for $\sin(A-B)$.

Assuming the identity for $\sin(A+B)$ is correct for all values of *A* and *B*, prove a similar result for $\sin(A-B)$.

[3 marks]

AQA 2018 Paper 2 Question 8:

8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

8 (b) (i) Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

8 (b) (ii) Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

AQA Sample Paper 2 Question 5:

5 (a) Determine a sequence of transformations which maps the graph of $y = \cos\theta$ onto the graph of $y = 3\cos\theta + 3\sin\theta$

Fully justify your answer.

[6 marks]

5 (b) Hence or otherwise find the least value and greatest value of

$$4 + (3\cos\theta + 3\sin\theta)^2$$

Fully justify your answer.

[3 marks]

Edexcel 2018 Paper 2 Question 7:

7. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Edexcel 2018 Paper 2 Question 12:

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta\sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)