



A Level Mathematics Year 2 Exam Questions by Topic

Chapter 17: Statistical hypothesis testing

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: <https://amzn.to/39umfr5> and <https://amzn.to/3hE8kBL>). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

<https://mathsaurus.com/>

OCR B MEI 2018 Paper 2 Question 13:

- 13** Each weekday Keira drives to work with her son Kaito. She always sets off at 8.00 a.m. She models her journey time, x minutes, by the distribution $X \sim N(15, 4)$.

Over a long period of time she notes that her journey takes less than 14 minutes on 7% of the journeys, and takes more than 18 minutes on 31% of the journeys.

- (i) Investigate whether Keira's model is a good fit for the data. [3]

Kaito believes that Keira's value for the variance is correct, but realises that the mean is not correct.

- (ii) Find, correct to two significant figures, the value of the mean that Keira should use in a refined model which does fit the data. [2]

Keira buys a new car. After driving to work in it each day for several weeks, she randomly selects the journey times for n of these days. Her mean journey time for these n days is 16 minutes. Using the refined model she conducts a hypothesis test to see if her mean journey time has changed, and finds that the result is significant at the 5% level.

- (iii) Determine the smallest possible value of n . [5]
-

AQA Sample Paper 3 Question 14:

- 14** A survey during 2013 investigated mean expenditure on bread and on alcohol.
The 2013 survey obtained information from 12 144 adults.
The survey revealed that the mean expenditure per adult per week on bread was 127p.
- 14 (a)** For 2012, it is known that the expenditure per adult per week on bread had mean 123p, and a standard deviation of 70p.
- 14 (a) (i)** Carry out a hypothesis test, at the 5% significance level, to investigate whether the mean expenditure per adult per week on bread changed from 2012 to 2013.
Assume that the survey data is a random sample taken from a normal distribution.
[5 marks]
- 14 (a) (ii)** Calculate the greatest and least values for the sample mean expenditure on bread per adult per week for 2013 that would have resulted in acceptance of the null hypothesis for the test you carried out in part (a)(i).
Give your answers to two decimal places.
[2 marks]
- 14 (b)** The 2013 survey revealed that the mean expenditure per adult, per week on alcohol was 324p.
The mean expenditure per adult per week on alcohol for 2009 was 307p.
A test was carried out on the following hypotheses relating to mean expenditure per adult per week on alcohol in 2013.
 $H_0 : \mu = 307$
 $H_1 : \mu \neq 307$
This test resulted in the null hypothesis, H_0 , being rejected.
State, with a reason, whether the test result supports the following statements:
- 14 (b) (i)** the mean UK expenditure on alcohol per adult per week increased by 17p from 2009 to 2013;
[2 marks]
- 14 (b) (ii)** the mean UK consumption of alcohol per adult per week changed from 2009 to 2013.
[2 marks]
-

Edexcel 2018 Paper 3 Question 2:

2. Tessa owns a small clothes shop in a seaside town. She records the weekly sales figures, £ w , and the average weekly temperature, t °C, for 8 weeks during the summer. The product moment correlation coefficient for these data is -0.915

- (a) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the correlation between sales figures and average weekly temperature is negative. (3)
- (b) Suggest a possible reason for this correlation. (1)

Tessa suggests that a linear regression model could be used to model these data.

- (c) State, giving a reason, whether or not the correlation coefficient is consistent with Tessa's suggestion. (1)
- (d) State, giving a reason, which variable would be the explanatory variable. (1)

Tessa calculated the linear regression equation as $w = 10\,755 - 171t$

- (e) Give an interpretation of the gradient of this regression equation. (1)

Edexcel 2018 Paper 3 Question 5:

5. The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

- (a) Find the probability that a randomly selected battery will last for longer than 16 hours. (1)

At the start of her exams Alice put 4 new batteries in her calculator. She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

- (b) Find the probability that her calculator will not stop working for Alice's remaining exams. (5)

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

- (c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures. (3)

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

- (d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief. (5)

Edexcel Sample Paper 3 Question 2:

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w km, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

- (a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C (1)
- (b) State what is measured by the product moment correlation coefficient. (1)
- (c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero. (3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$

- (d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics. (1)

Edexcel Sample Paper 3 Question 3:

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

- (a) find the probability that a randomly chosen strip of metal **can** be used. (5)

Ten strips of metal are selected at random.

- (b) Find the probability fewer than 4 of these strips **cannot** be used. (2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

- (c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm (5)

OCR A Sample Paper 2 Question 10:

- 10** In the past the time, in minutes, spent by customers in a certain library had mean 32.5 and standard deviation 8.2. Following a change of layout in the library, the mean time spent in the library by a random sample of 50 customers is found to be 34.5 minutes. Assuming that the standard deviation remains at 8.2, test at the 5% significance level whether the mean time spent by customers in the library has changed. [7]

AQA 2018 Paper 3 Question 18:

- 18** In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.
- Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g
- 18 (a)** After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.
- 18 (a) (i)** State the sampling method used to collect the survey. [1 mark]
- 18 (a) (ii)** Explain why this sample should not be used to conduct a hypothesis test. [1 mark]
- 18 (b)** A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g
- Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.
- Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation. [6 marks]

AQA Sample Paper 3 Question 10:

- 10** Shona calculated four correlation coefficients using data from the Large Data Set.
- In each case she calculated the correlation coefficient between the masses of the cars and the CO₂ emissions for varying sample sizes.
- A summary of these calculations, labelled A to D, are listed in the table below.

	Sample size	Correlation coefficient
A	3827	0.088
B	3735	0.246
C	24	0.400
D	1250	- 1.183

Shona would like to use calculation A to test whether there is evidence of positive correlation between mass and CO₂ emissions.

She finds the critical value for a one-tailed test at the 5% level for a sample of size 3827 is 0.027

- 10 (a) (i)** State appropriate hypotheses for Shona to use in her test. **[1 mark]**
- 10 (a) (ii)** Determine if there is sufficient evidence to reject the null hypothesis.
Fully justify your answer. **[1 mark]**
- 10 (b)** Shona's teacher tells her to remove calculation D from the table as it is incorrect.
Explain how the teacher knew it was incorrect. **[1 mark]**
- 10 (c)** Before performing calculation B, Shona cleaned the data. She removed all cars from the Large Data Set that had incorrect masses.
Using your knowledge of the large data set, explain what was incorrect about the masses which were removed from the calculation. **[1 mark]**
- 10 (d)** Apart from CO₂ and CO emissions, state one other type of emission that Shona could investigate using the Large Data Set. **[1 mark]**
- 10 (e)** Wesley claims that calculation C shows that a heavier car causes higher CO₂ emissions.
Give **two** reasons why Wesley's claim may be incorrect. **[2 marks]**
-

Edexcel Sample Paper 3 Question 5:

5. A company sells seeds and claims that 55% of its pea seeds germinate.

- (a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce. (1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

- (b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate. (3)

- (c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution. (1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

- (d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate. (3)

- (e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55% (1)

OCR A 2018 Paper 2 Question 10:

10 A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14781049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

- (i) Explain why Dipak omitted the number 810 from his list. [1]
- (ii) Explain why Dipak's sample is not random. [1]

The mean height of all trees of this species is known to be 4.2m. Dipak wishes to test whether the mean height of trees in the forest is less than 4.2m. He now uses a correct method to choose a random sample of 50 trees and finds that their mean height is 4.0m. It is given that the standard deviation of trees in the forest is 0.8m.

- (iii) Carry out the test at the 2% significance level. [7]

OCR A Sample Paper 2 Question 12:

- 12 The table shows information for England and Wales, taken from the UK 2011 census.

Total population	Number of children aged 5-17
56 075 912	8 473 617

A random sample of 10 000 people in another country was chosen in 2011, and the number, m , of children aged 5-17 was noted. It was found that there was evidence at the 2.5% level that the proportion of children aged 5-17 in the same year was higher than in the UK. Unfortunately, when the results were recorded the value of m was omitted. Use an appropriate normal distribution to find an estimate of the smallest possible value of m . [5]

OCR B MEI Sample Paper 2 Question 9:

- 9 A geyser is a hot spring which erupts from time to time. For two geysers, the duration of each eruption, x minutes, and the waiting time until the next eruption, y minutes, are recorded.
- (i) For a random sample of 50 eruptions of the first geyser, the correlation coefficient between x and y is 0.758. The critical value for a 2-tailed hypothesis test for correlation at the 5% level is 0.279. Explain whether or not there is evidence of correlation in the population of eruptions. [2]

The scatter diagram in Fig. 9 shows the data from a random sample of 50 eruptions of the second geyser.

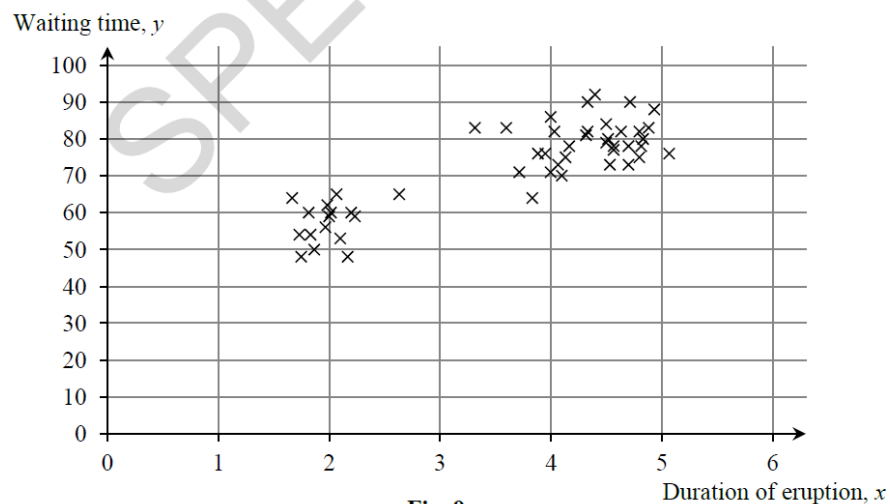


Fig. 9

- (ii) Stella claims the scatter diagram shows evidence of correlation between duration of eruption and waiting time. Make two comments about Stella's claim. [2]