MATHSAURUS

A Level Mathematics Year 2 Exam Questions by Topic
Chapter 14: Numerical methods

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

OCR A Sample Paper 3 Question 2:

2 **(i)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx.$$ **[3]**

(ii) Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(i)**. **[1]**

OCR B MEI 2018 Paper 1 Question 2:

2 By considering a change of sign, show that the equation $e^x - 5x^3 = 0$ has a root between 0 and 1. **[2]**

OCR B MEI 2018 Paper 1 Question 13:

13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

(ii) Find the first three terms of the binomial expansion of $f(x)$. **[3]**

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. **[3]**

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

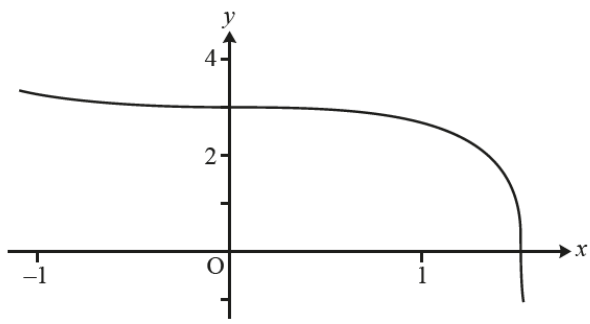


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

OCR B MEI Sample Paper 1 Question 6:

6 Fig. 6 shows a partially completed spreadsheet which uses the trapezium rule with four strips to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\, dx$.

	A	B	C	D	E
1		x	sin x	y	
2	0	0.0000	0.0000	1.0000	0.5000
3	0.125	0.3927	0.3827	1.1759	1.1759
4	0.25	0.7854	0.7071	1.3066	1.3066
5	0.375	1.1781	0.9239	1.3870	1.3870
6	0.5	1.5708	1.0000	1.4142	0.7071
7					5.0766
8					

Fig. 6

(i) Show how the value in cell B3 is calculated. **[1]**

(ii) Show how the value in cell E7 is calculated from the values in cells D2 to D6. **[1]**

(iii) Complete the calculation to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1+\sin x}\, dx$, giving the answer to 3 significant figures. **[2]**

OCR B MEI Sample Paper 3 Question 10:

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

(i) Show that $x = -1$ is a root of $f(x) = 0$. **[1]**

(ii) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. **[2]**

(iii) Show that $f(x) = (x+1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. **[3]**

(iv) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. **[1]**

(v) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x) = 0$ may be written

$$x_{n+1} = \frac{2x_n^{\ 3} + 2}{3x_n^{\ 2} - 2}.$$

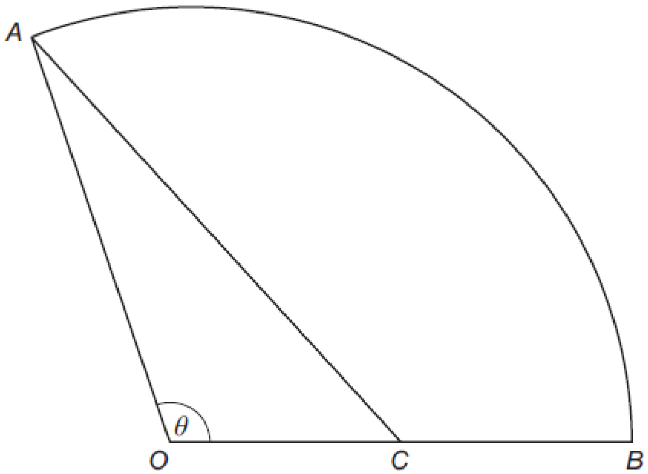
Determine the root of $g(x) = 0$ which lies between $x = 1$ and $x = 2$ correct to 4 significant figures. **[3]**

AQA 2018 Paper 1 Question 8:

8 The diagram shows a sector of a circle *OAB*.

C is the midpoint of *OB*.

Angle *AOB* is θ radians.



8 (a) Given that the area of the triangle *OAC* is equal to one quarter of the area of the sector *OAB*, show that $\theta = 2\sin\theta$

[4 marks]

8 (b) Use the Newton-Raphson method with $\theta_1 = \pi$, to find θ_3 as an approximation for θ. Give your answer correct to five decimal places.

[3 marks]

8 (c) Given that $\theta = 1.89549$ to five decimal places, find an estimate for the percentage error in the approximation found in part **(b)**.

[1 mark]

AQA 2018 Paper 1 Question 11:

11 The daily world production of oil can be modelled using

$$V = 10 + 100\left(\frac{t}{30}\right)^3 - 50\left(\frac{t}{30}\right)^4$$

where V is volume of oil in millions of barrels, and t is time in years since 1 January 1980.

11 (a) (i) The model is used to predict the time, T, when oil production will fall to zero.

Show that T satisfies the equation

$$T = \sqrt[3]{60T^2 + \frac{162\,000}{T}}$$

[3 marks]

11 (a) (ii) Use the iterative formula $T_{n+1} = \sqrt[3]{60{T_n}^2 + \frac{162\,000}{T_n}}$, with $T_0 = 38$, to find the values of T_1, T_2, and T_3, giving your answers to three decimal places.

[2 marks]

11 (a) (iii) Explain the relevance of using $T_0 = 38$

[1 mark]

11 (b) From 1 January 1980 the daily use of oil by one technologically developing country can be modelled as

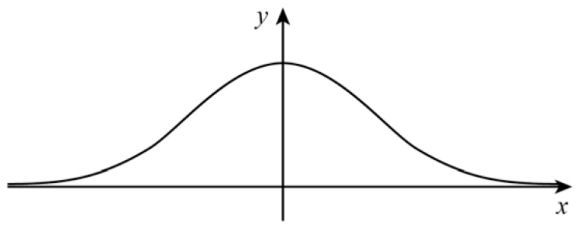
$$V = 4.5 \times 1.063^t$$

Use the models to show that the country's use of oil and the world production of oil will be equal during the year 2029.

[4 marks]

AQA Sample Paper 3 Question 7:

7 The diagram shows part of the graph of $y = e^{-x^2}$

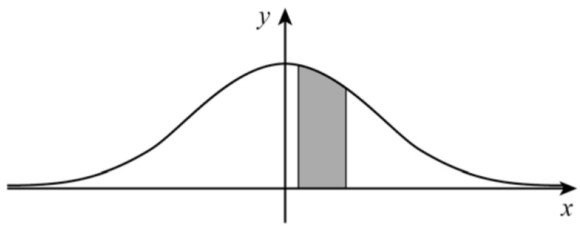


The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

7 (a) Find the values of x for which the graph is concave.

[4 marks]

7 (b) The finite region bounded by the x-axis and the lines x = 0.1 and x = 0.5 is shaded.



Use the trapezium rule, with 4 strips, to find an estimate for $\int_{0.1}^{0.5} e^{-x^2}\,dx$

Give your estimate to four decimal places.

[3 marks]

7 (c) Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

[2 marks]

7 (d) By considering the area of a rectangle, and using your answer to part **(b)**, prove that the shaded area is 0.4 correct to 1 decimal place.

[3 marks]

AQA Sample Paper 2 Question 4:

4 The equation $x^3 - 3x + 1 = 0$ has three real roots.

4 (a) Show that one of the roots lies between –2 and –1

[2 marks]

4 (b) Taking $x_1 = -2$ as the first approximation to one of the roots, use the Newton-Raphson method to find x_2, the second approximation.

[3 marks]

4 (c) Explain why the Newton-Raphson method fails in the case when the first approximation is $x_1 = -1$

[1 mark]

Edexcel 2018 Paper 1 Question 4:

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

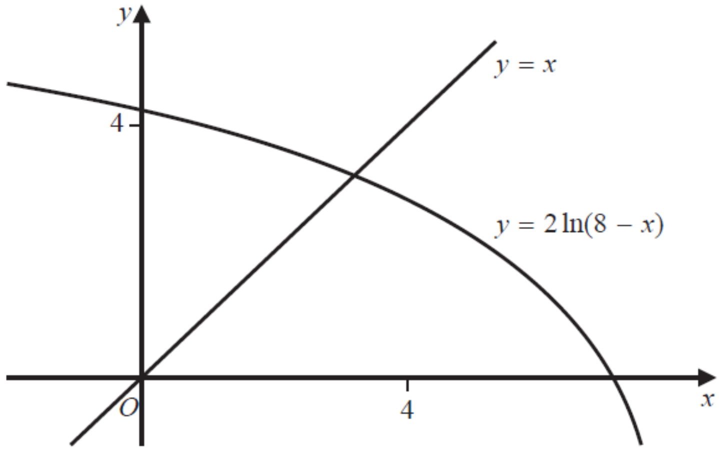


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

Edexcel 2018 Paper 2 Question 5:

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

Edexcel Sample Paper 1 Question 8:

8.

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

Edexcel Sample Paper 1 Question 14:

14.

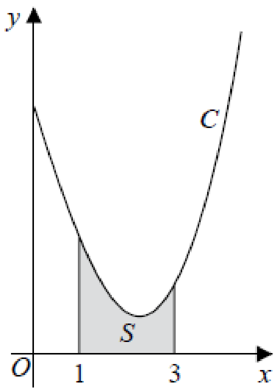


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

OCR A 2018 Paper 1 Question 2:

2 **(i)** Use the trapezium rule, with four strips each of width 0.5, to estimate the value of

$$\int_0^2 e^{x^2}\,dx$$

giving your answer correct to 3 significant figures. **[3]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

OCR A 2018 Paper 3 Question 5:

5 **(i)** Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{11}{4} - \sqrt{2}.$$ **[5]**

(ii) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx.$$ **[6]**

(iii) Using your answers to parts **(i)** and **(ii)**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. **[2]**

OCR A Sample Paper 1 Question 9:

9 The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\,3} - x_n^{\,2} - 10}{3x_n^{\,2} - 2x_n - 5}.$$

[3]

(ii) Apply the iterative formula in part **(i)** with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(iii) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(iv) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**