A Level Mathematics Year 2 Exam Questions by Topic
Chapter 13: Differential equations

These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: `https://amzn.to/39umfr5` and `https://amzn.to/3hE8kBL`). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

`https://mathsaurus.com/`

OCR B MEI Sample Paper 2 Question 14:

14 In a chemical reaction, the mass m grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{m}{t(1+2t)} \ .$$

At time 1 minute, the mass of the chemical is 1 gram.

(i) Solve the differential equation to show that $m = \dfrac{3t}{(1+2t)}$. **[8]**

(ii) Hence

(A) find the time when the mass is 1.25 grams, **[2]**

(B) show what happens to the mass of the chemical as t becomes large. **[2]**

AQA 2018 Paper 2 Question 9:

9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8-t)$$

[3 marks]

9 (b) Hence, show that

$$x^2 = 4032t(16-t)$$

[3 marks]

9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

AQA Sample Paper 1 Question 6:

6 Sam goes on a diet. He assumes that his mass, m kg after t days, decreases at a rate that is inversely proportional to the cube root of his mass.

6 (a) Construct a differential equation involving m , t and a positive constant k to model this situation.

[3 marks]

6 (b) Explain why Sam's assumption may not be appropriate.

[1 mark]

AQA Sample Paper 1 Question 15:

15 The height x metres, of a column of water in a fountain display satisfies the differential equation $\frac{dx}{dt} = \frac{8\sin 2t}{3\sqrt{x}}$, where t is the time in seconds after the display begins.

15 (a) Solve the differential equation, given that initially the column of water has zero height.

Express your answer in the form $x = f(t)$

[7 marks]

15 (b) Find the maximum height of the column of water, giving your answer to the nearest cm.

[1 mark]

OCR B MEI 2018 Paper 2 Question 17:

17 **(i)** Express $\frac{(x^2 - 8x + 9)}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Express y in terms of x given that

$$\frac{dy}{dx} = \frac{y(x^2 - 8x + 9)}{(x+1)(x-2)^2} \text{ and } y = 16 \text{ when } x = 3.$$

[7]

AQA Sample Paper 3 Question 3:

3 A circular ornamental garden pond, of radius 2 metres, has weed starting to grow and cover its surface.

As the weed grows, it covers an area of A square metres. A simple model assumes that the weed grows so that the rate of increase of its area is proportional to A.

3 (a) Show that the area covered by the weed can be modelled by

$$A = Be^{kt}$$

where B and k are constants and t is time in days since the weed was first noticed.

[4 marks]

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]

3 (c) State one limitation of the model.

[1 mark]

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

Edexcel 2018 Paper 1 Question 10:

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

Edexcel 2018 Paper 2 Question 10:

10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)

(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.

(2)

(c) Suggest a limitation of the model.

(1)

Edexcel Sample Paper 2 Question 16:

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

OCR A 2018 Paper 1 Question 13:

13 A scientist is attempting to model the number of insects, N, present in a colony at time t weeks. When $t = 0$ there are 400 insects and when $t = 1$ there are 440 insects.

(i) A scientist assumes that the rate of increase of the number of insects is inversely proportional to the number of insects present at time t.

(a) Write down a differential equation to model this situation. **[1]**

(b) Solve this differential equation to find N in terms of t. **[4]**

(ii) In a revised model it is assumed that $\dfrac{dN}{dt} = \dfrac{N^2}{3988e^{0.2t}}$. Solve this differential equation to find N in terms of t. **[6]**

(iii) Compare the long-term behaviour of the two models. **[2]**

OCR A 2018 Paper 3 Question 7:

7 The gradient of the curve $y = f(x)$ is given by the differential equation

$$(2x-1)^3\frac{dy}{dx} + 4y^2 = 0$$

and the curve passes through the point $(1, 1)$. By solving this differential equation show that

$$f(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1},$$

where a and b are integers to be determined. **[9]**

OCR A Sample Paper 1 Question 14:

14 John wants to encourage more birds to come into the park near his house. Each day, starting on day 1, he puts bird food out and then observes the birds for one hour. He records the maximum number of birds that he observes at any given moment in the park each day. He believes that his observations may be modelled by the differential equation

$$\frac{dn}{dt} = 0.1n\left(1 - \frac{n}{50}\right)$$

where n is the maximum number of birds that he observed at any given moment on day t.

(i) Show that the general solution to the differential equation can be written in the form $n = \frac{50A}{e^{-0.1t} + A}$, where A is an arbitrary positive constant. **[9]**

(ii) Using his model, determine the maximum number of birds that John would expect to observe at any given moment in the long term. **[1]**

(iii) Write down one possible refinement of this model. **[1]**

(iv) Write down one way in which John's model is not appropriate. **[1]**

OCR A Sample Paper 2 Question 6:

6 Helga invests £4000 in a savings account. After t days, her investment is worth £y. The rate of increase of y is ky, where k is a constant.

(i) Write down a differential equation in terms of t, y and k. **[1]**

(ii) Solve your differential equation. Hence find the value of Helga's investment after t days. Give your answer in terms of k. **[4]**

It is given that $k = \frac{1}{365}\ln\left(1 + \frac{r}{100}\right)$ where r% is the rate of interest per annum. During the first year the rate of interest is 6% per annum.

(iii) Find the value of Helga's investment after 90 days. **[2]**

After one year (365 days), the rate of interest drops to 5% per annum.

(iv) Find the total time that it will take for Helga's investment to double in value. **[5]**