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**A Level Mathematics Year 2 Exam Questions by Topic**  
**Chapter 10: Integration**

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These questions are taken from the Specimen Exam materials and the real 2018 papers for the new syllabus AS and A-level mathematics courses and arranged by chapter of the textbooks by Goldie et al (available here: <https://amzn.to/39umfr5> and <https://amzn.to/3hE8kBL> ). There are a mixture of questions from OCR A, OCR B (MEI), Edexcel and AQA. Although the style of questions varies a little across the exam boards the content of the syllabus is almost identical so these are suitable for students preparing for any exam board.

Free problem sets for all other chapters, as well as video solutions, full past papers and other content for GCSE and A-level maths can be found at:

<https://mathsaurus.com/>

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OCR A Sample Paper 3 Question 7:

7 (i) Find  $\int 5x^3\sqrt{x^2+1} \, dx$ . [5]

(ii) Find  $\int \theta \tan^2 \theta \, d\theta$ . You may use the result  $\int \tan \theta \, d\theta = \ln|\sec \theta| + c$ . [5]

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OCR B MEI 2018 Paper 1 Question 8:

8 (i) Show that  $8 \sin^2 x \cos^2 x$  can be written as  $1 - \cos 4x$ . [3]

(ii) Hence find  $\int \sin^2 x \cos^2 x \, dx$ . [3]

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OCR B MEI 2018 Paper 1 Question 10:

10 Fig. 10 shows the graph of  $y = (k-x)\ln x$  where  $k$  is a constant ( $k > 1$ ).

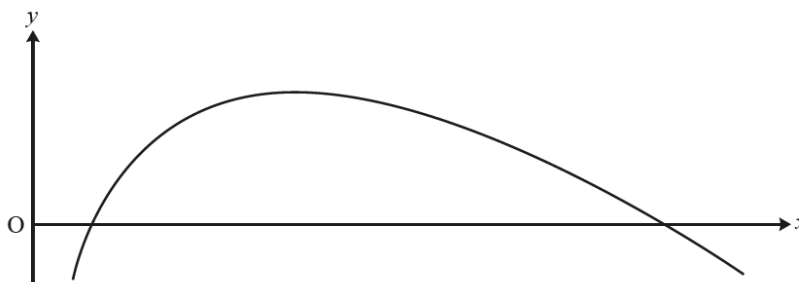


Fig. 10

Find, in terms of  $k$ , the area of the finite region between the curve and the  $x$ -axis. [8]

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OCR B MEI Sample Paper 1 Question 8:

8 Find  $\int x^2 e^{2x} dx$ . [7]

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OCR B MEI Sample Paper 2 Question 3:

3 Evaluate  $\int_0^{\frac{\pi}{12}} \cos 3x dx$ , giving your answer in exact form. [3]

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OCR B MEI Sample Paper 2 Question 13:

13 Evaluate  $\int_0^1 \frac{1}{1+\sqrt{x}} dx$ , giving your answer in the form  $a + b \ln c$ , where  $a$ ,  $b$  and  $c$  are integers. [6]

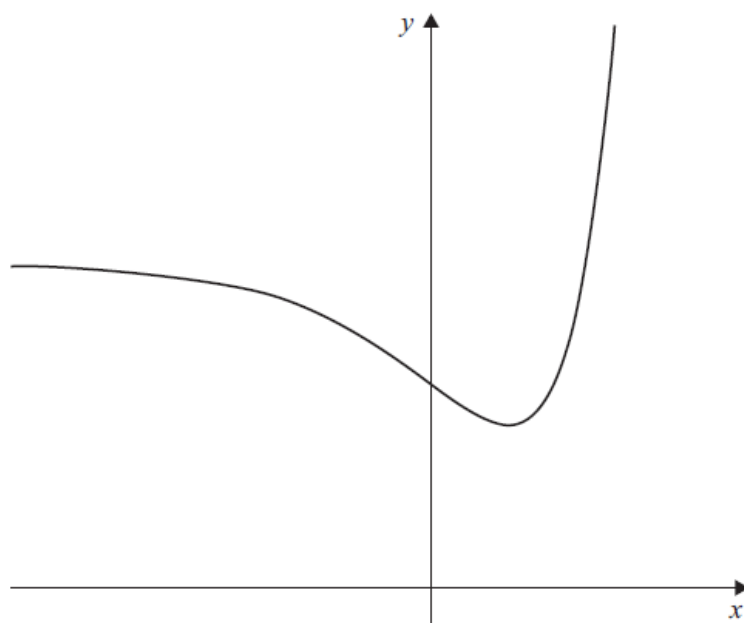
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AQA 2018 Paper 2 Question 7:

- 7 A function  $f$  has domain  $\mathbb{R}$  and range  $\{y \in \mathbb{R} : y \geq e\}$

The graph of  $y = f(x)$  is shown.



The gradient of the curve at the point  $(x, y)$  is given by  $\frac{dy}{dx} = (x - 1)e^x$

Find an expression for  $f(x)$ .

Fully justify your answer.

[8 marks]

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AQA Sample Paper 3 Question 4:

- 4  $\int_1^2 x^3 \ln(2x) dx$  can be written in the form  $p \ln 2 + q$ , where  $p$  and  $q$  are rational numbers.

Find  $p$  and  $q$ .

[5 marks]

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AQA Sample Paper 3 Question 6:

- 6 Find the value of  $\int_1^2 \frac{6x + 1}{6x^2 - 7x + 2} dx$ , expressing your answer in the form

$m \ln 2 + n \ln 3$ , where  $m$  and  $n$  are integers.

[8 marks]

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AQA 2018 Paper 3 Question 8:

**8 (a)** Prove the identity  $\frac{\sin 2x}{1 + \tan^2 x} \equiv 2 \sin x \cos^3 x$  **[3 marks]**

**8 (b)** Hence find  $\int \frac{4 \sin 4\theta}{1 + \tan^2 2\theta} d\theta$  **[6 marks]**

**8 (b)** Find the exact value of  $\int_0^1 2^x \sqrt{3 + 2^x} dx$   
Fully justify your answer. **[6 marks]**

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Edexcel 2018 Paper 1 Question 7:

- 7.** Given that  $k \in \mathbb{Z}^+$
- (a) show that  $\int_k^{3k} \frac{2}{(3x - k)} dx$  is independent of  $k$ , **(4)**
- (b) show that  $\int_k^{2k} \frac{2}{(2x - k)^2} dx$  is inversely proportional to  $k$ . **(3)**

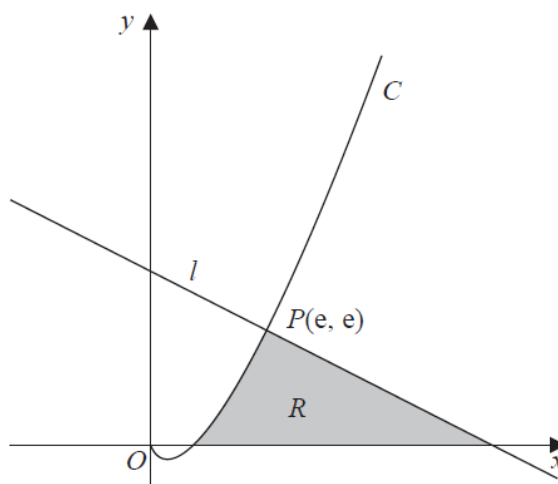
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Edexcel 2018 Paper 1 Question 13:

**13.** Show that 
$$\int_0^2 2x\sqrt{x+2} dx = \frac{32}{15}(2 + \sqrt{2})$$
 **(7)**

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13.



**Figure 2**

Figure 2 shows a sketch of part of the curve  $C$  with equation  $y = x \ln x$ ,  $x > 0$

The line  $l$  is the normal to  $C$  at the point  $P(e, e)$

The region  $R$ , shown shaded in Figure 2, is bounded by the curve  $C$ , the line  $l$  and the  $x$ -axis.

Show that the exact area of  $R$  is  $Ae^2 + B$  where  $A$  and  $B$  are rational numbers to be found.

**(10)**

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Edexcel Sample Paper 1 Question 4:

4. Given that  $a$  is a positive constant and

$$\int_a^{2a} \frac{t+1}{t} dt = \ln 7$$

show that  $a = \ln k$ , where  $k$  is a constant to be found.

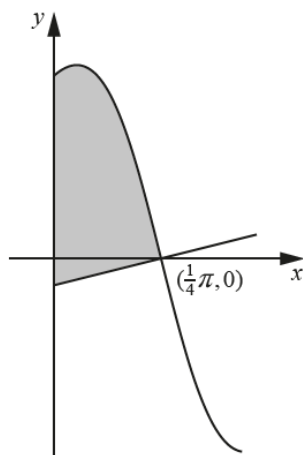
**(4)**

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OCR A 2018 Paper 1 Question 12:

12 In this question you must show detailed reasoning.



The diagram shows the curve  $y = \frac{4 \cos 2x}{3 - \sin 2x}$ , for  $x \geq 0$ , and the normal to the curve at the point  $(\frac{1}{4}\pi, 0)$ . Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the  $y$ -axis is  $\ln \frac{9}{4} + \frac{1}{128}\pi^2$ . [10]

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OCR A Sample Paper 2 Question 4:

4 (i) Express  $\frac{1}{(x-1)(x+2)}$  in partial fractions. [2]

(ii) In this question you must show detailed reasoning.

Hence find  $\int_2^3 \frac{1}{(x-1)(x+2)} dx$ . Give your answer in its simplest form. [5]

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