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# A Level Mathematics B (MEI)

## H640/03 Pure Mathematics and Comprehension

Sample Insert

## Date – Morning/Afternoon

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**Archimedes's approximation of $\pi$**

The constant $\pi$ is defined to be the circumference of a circle divided by its diameter.

The value of $\pi$ has been determined to an accuracy of more than twelve trillion decimal places. To the non-mathematician this may appear strange since it is not possible to measure the circumference and diameter of a circle to that degree of accuracy; this article explains how one of the greatest mathematicians of all time found the value of $\pi$ to a high degree of accuracy without requiring any physical measurement.

Archimedes (287-212 BC) lived in Syracuse, Sicily. He developed many branches of mathematics, including calculus, in which he devised methods for finding areas under parabolas nearly 2000 years before Newton and Leibniz, and mechanics, in which he found the centres of gravity of various plane figures and solids and devised a method for calculating the weight of a body immersed in a liquid.

Whilst absorbed in a mathematical problem, Archimedes was killed by a soldier during the capture of Syracuse by the Romans.

Archimedes's method for determining the value of $\pi$ is described below.

Fig. C1 shows a circle with unit radius and two regular hexagons.

The smaller regular hexagon has its vertices on the circle; it is called an *inscribed* polygon. Its perimeter is 6.

The larger regular hexagon has the midpoints of its edges on the circle; it is called an *escribed* polygon. Its perimeter is $4\sqrt{3}$.



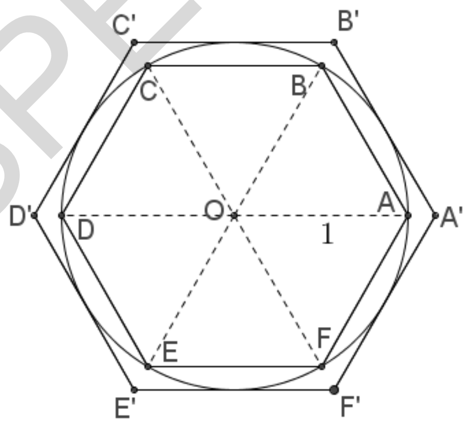


**Fig. C1**

The circumference of the circle is greater than the perimeter, ABCDEF, of the smaller hexagon but less than the perimeter, A'B'C'D'E'F', of the larger hexagon. Dividing the perimeters by the diameter of the circle gives lower and upper bounds for $\pi$ of 3 and $2\sqrt{3}$, so that $3 < \pi < 2\sqrt{3}$.

To find tighter bounds, Archimedes repeatedly doubled the number of edges in the two regular polygons, from 6 to 12, 24, 48 and finally 96. The process of doubling the number of edges is described below.

Fig. C2 shows two adjacent vertices, P and Q, of a regular polygon inscribed in a circle with unit radius and centre O. PQ has length *a*. M is the midpoint of PQ. OM is extended to meet the circle at R. MR has length *h*. PR and RQ are adjacent edges of a regular polygon which has twice as many edges as the polygon which has PQ as an edge. PR has length *b*.

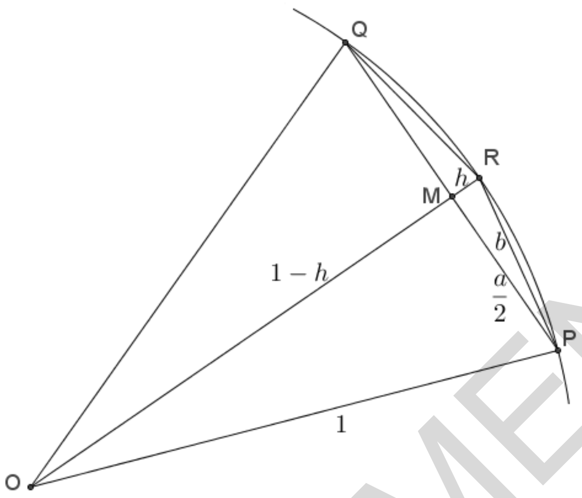


**Fig. C2**

Applying Pythagoras' Theorem

- to triangle OMP gives $1=\frac{a^2}{4}+(1-h)^2$,
- to triangle PMR gives $b^2=\frac{a^2}{4}+h^2$.

For the inscribed regular hexagon, $a=1$. Substituting $a=1$ in the equations above gives $h=\frac{2-\sqrt{3}}{2}$ and $b=\sqrt{2-\sqrt{3}}$; this can be written in the equivalent form $b=\frac{\sqrt{6}-\sqrt{2}}{2}$. Therefore a regular polygon with 12 edges inscribed in a unit circle has edge length $\frac{\sqrt{6}-\sqrt{2}}{2}$.

Archimedes repeated this process to find the edge lengths of inscribed regular polygons with 24, 48 and 96 edges, and then used a similar technique for escribed regular polygons.

The inscribed and escribed regular polygons with 96 edges provide bounds for $\pi$ which we now write, using decimal notation, as

$$3.14103... < \pi < 3.14271... .$$

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