OCR
RECOGNISING ACHIEVEMENT

Wednesday 30 May 2012 – Afternoon

FSMQ ADVANCED LEVEL

6993 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

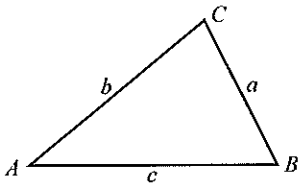
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 (i) Find the range of values of x satisfying $x^2 - 4x + 3 \leqslant 0$. [3]

(ii) Show this range on the number line provided. [1]

2 A die has 6 faces numbered one to six. The die is biased so that when it is thrown the probability of obtaining a six is $\frac{1}{5}$.

The die is thrown 5 times.

Find the probability of obtaining

(i) at least 1 six, [2]

(ii) exactly 3 sixes. [4]

3 The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.

(i) Show that the value of a is -7. [2]

(ii) Factorise $f(x)$. [3]

4 A car moves from rest with constant acceleration on a straight road. When the car passes a point A it is travelling at $10\,\text{m}\,\text{s}^{-1}$ and when it passes a point B further along the road it is travelling at $16\,\text{m}\,\text{s}^{-1}$.

The car takes 10 seconds to travel from A to B.

Find

- the distance AB,
- the constant acceleration. [4]

5 (i) Show that the equation $3\cos^2\theta = \sin\theta + 1$ can be written as $3\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

(ii) Solve this equation to find values of θ in the range $0° < \theta < 360°$ that satisfy

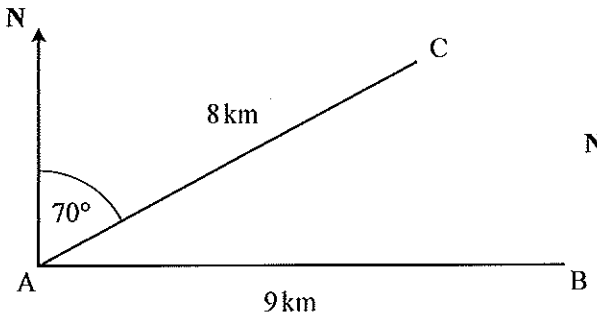
$$3\cos^2\theta = \sin\theta + 1 .$$ **[4]**

6 The equation of a curve is $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the curve has a stationary point where $x = 2$. **[4]**

(ii) Determine whether the stationary value where $x = 2$ is a maximum or minimum. **[2]**

7 A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A.
Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



Not to scale

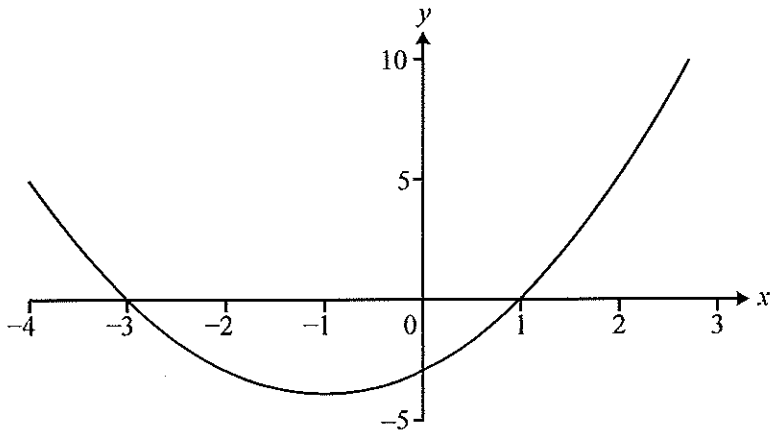
Calculate

(i) the distance he is now from port B, **[3]**

(ii) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. **[4]**

8 (i) Show that $\int_0^2 (x^2 + 2x - 3)\,dx = \frac{2}{3}$. [3]

The diagram shows part of the curve $y = x^2 + 2x - 3$.



(ii) Marc claims that the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.

Explain why he is wrong. [1]

(iii) Calculate the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$. [3]

9 The height above the ground of a seat on a fairground big wheel is h metres. At time t **minutes** after the wheel starts, h is given by

$$h = 7 - 5\cos(480t)°.$$

(i) Write down the initial height above the ground of the seat (when $t = 0$). [1]

(ii) Find the greatest height reached by the seat. [2]

(iii) Calculate the time of the first occasion when the seat is 9 metres above the ground.
Give your answer correct to the nearest second. [4]

Section B

10 A (1, 10), B (8, 9) and C (7, 2) are three points.

(i) Find the coordinates of the midpoint, M, of AC. **[1]**

(ii) Find the equation of the circle with AC as diameter. **[4]**

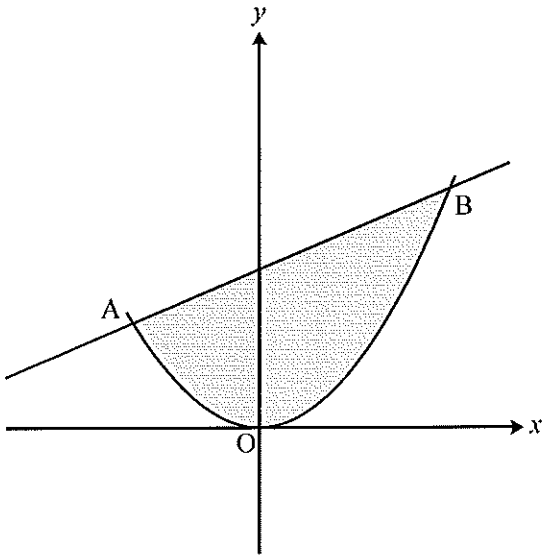
(iii) Show that B lies on this circle. **[1]**

(iv) Prove that AM and BM are perpendicular. **[3]**

(v) BD is a diameter of this circle. Find the coordinates of D. **[3]**

11 The shaded region in the diagram shows a wooden shape.

The curve has equation $y = \frac{1}{2}x^2$ and the coordinates of A are $(-2, 2)$.



The line AB is the normal to the curve at the point A.

(i) Find the equation of the line AB. **[5]**

(ii) Find the coordinates of the point B where the line AB meets the curve again. **[3]**

(iii) Find the shaded area. **[4]**

12 The Highway Code gives a table of shortest stopping distances (d feet) for a vehicle travelling at v miles per hour.

The formula used for this table is given by

$$d = av^2 + bv\,.$$

Two entries in the table are given below.

v mph	d feet
30	75
60	240

(i) By forming and solving a pair of simultaneous equations in a and b, show that the formula is

$$d = \frac{v^2}{20} + v.$$ **[5]**

(ii) Find the difference between the stopping distances for a car travelling at 65 mph and a car travelling at 70 mph. **[3]**

(iii) Many drivers maintain a distance of 50 feet or less when driving on a motorway.

Use the formula in part **(i)** to find the speed at which the shortest stopping distance is 50 feet. **[4]**

Question 13 is printed overleaf

13 **(i)** Find the coefficients a, b and c in the expansion

$$(2+h)^3 = 8 + ah + bh^2 + ch^3 .$$ **[3]**

(ii) The graph of the equation $y = x^3$ passes through the points P and Q which have x-coordinates 2 and $2 + h$ respectively.

Show that the gradient of the chord PQ is $\dfrac{(2+h)^3 - 8}{h}$. **[3]**

(iii) Express $\dfrac{(2+h)^3 - 8}{h}$ as a quadratic function of h. **[2]**

(iv) As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

Deduce the value of the gradient of the tangent at P. **[1]**

(v) Kareen uses the same method to deduce the value of the gradient of the tangent at the point (2, 16) on the curve $y = x^4$.

The first three lines of her working are given below and in the answer booklet.

Take P to be the point (2, 16)

Take Q to be the point $(2 + h, (2 + h)^4)$

The gradient of the chord PQ is given by $\dfrac{(2+h)^4 - 16}{h} =$

Complete Kareen's working. **[3]**